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Identical price categories in oligopolistic markets. Innocent behaviour or collusive practice?

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Temi e problemi
11

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Identical price categories in oligopolistic markets. Innocent behaviour or collusive practice?

Abstract

In many sectors competing firms choose the same (or similar) price categories - a price category being defined as a situation where a firm commits to sell a given set of products (or to supply a given set of markets), at a single price. In this Report we show that firms *might* choose homogeneous price categories in order to facilitate collusion. The reason being that similar price categories might make the enforcement of a given collusive outcome more likely; moreover, they may facilitate coordination on a given collusive outcome simpler; and they may make the market more transparent from the sellers' side, thereby facilitating reciprocal monitoring. However, homogeneous price categories might simply result from non-collusive competitive interaction. Specifically, firms will tend to adopt similar price categories because they face similar demand or cost conditions. Historical reasons and habits, as well as the design of efficient incentive schemes for employees represent other innocent motives for homogeneous price categories. Finally, the choice of the same price categories as rivals, by making the market more transparent from the demand side, may make a firm's aggressive pricing strategy more effective. Hence, our study suggests that adopting the same (or similar) categories should not be considered *per se* illegal, nor competition authorities should devote extra-effort to investigate sectors characterized by similar price categories. It is only an *explicit* agreement among firms to use homogeneous price categories that should be considered unlawful.

JEL Classification: K21, L13, L41
Keywords: Price discrimination, Oligopoly, Collusion, Competition Policy

Sintesi

In diversi settori le imprese scelgono categorie di prezzo uguali o simili – dove per categoria di prezzo si definisce l'impegno di un'impresa a vendere un dato insieme di prodotti (o di servire un dato insieme di mercati) ad un unico prezzo. La presente ricerca mostra che *è possibile* che le imprese scelgano categorie di prezzo simili con finalità pro-collusive. In particolare, la scelta di categorie di prezzo omogenee può rendere più facile il sostenimento di un accordo collusivo; può rendere più semplice il coordinamento su un dato esito collusivo e può rendere il mercato più trasparente dal lato dell'offerta, facilitando perciò il controllo del comportamento reciproco. D'altro canto la scelta di categorie di prezzo simili può essere semplicemente il risultato di interazione strategica non-collusiva tra le imprese. In particolare, le imprese possono adottare categorie di prezzo omogenee perché fronteggiano simili condizioni di domanda e di costo. Ragioni storiche, abitudini, così come l'attuazione di schemi di incentivo efficienti per i dipendenti costituiscono altre motivazioni innocenti alla base di categorie di prezzo simili. Infine, scegliendo categorie di prezzo simili a quelle adottate dai rivali, un'impresa può aumentare la trasparenza del mercato dal lato della domanda e rendere più efficace una strategia di prezzo aggressiva. Perciò il nostro studio suggerisce che la scelta di categorie di prezzo omogenee non dovrebbe essere considerata illegale *per se*, né particolare attenzione dovrebbe essere dedicata dalle autorità di tutela della concorrenza ai settori caratterizzati da categorie di prezzo omogenee. E' solo l'accordo *esplicito* tra le imprese al fine di adottare categorie di prezzo omogenee che dovrebbe essere considerato lesivo della concorrenza.

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Executive Summary

There exist several industries where competing firms choose the *same* (or homogenous) price categories, a *price category* being defined as a situation where a firm commits to sell a given set of products (or to supply a given set of markets), at a single price. (Note that the concept of homogeneity of price categories does not necessarily imply that firms also choose the same price levels for each price category.)

Examples of homogenous price categories include airlines that define business and leisure fares in the same way, and offer discounted fares according to the same age groups; record companies that sell hundreds of records grouped in the same four price categories; cinemas that offer discounted tickets the same day of the week; telephone companies that use the same definition of peak and off-peak times of calls, and so on.

We show in this Report that the existence of the same price categories *might* be the result of firms' coordination. Indeed, firms might have an incentive to choose homogeneous price categories in order to facilitate collusion. This is because committing to the same price categories allows firms to achieve the highest collusive profits, it makes deviation less profitable and the punishment of deviations harsher, thereby rendering collusion more likely. Moreover, homogeneous price categories may help firms solve coordination problems (it is easier to find an agreement - or to renegotiate it - when categories are defined in the same way) and may make the market more transparent on the sellers' side, facilitating reciprocal monitoring and thus again making collusion easier to sustain.

However, we also show that homogenous price categories might also be the product of normal (in the sense of non-collusive) competitive interactions in oligopolistic markets. In particular, firms which behave non-cooperatively and face similar demand or cost conditions will tend to use similar price categories even in an environment (such as one-shot market interaction) where collusion is impossible.

Furthermore, choosing exactly the same price categories might well be due to other innocent motives, such as historical reasons and habits (newcomers might find it easier to define price categories in the same way as incumbents; managers might prefer to continue to work with the same definitions of price categories; buyers might show hostility to changes in the definitions of input price categories, which might oblige them to modify categories of their own products), as well as the design of efficient incentive schemes for employees (a firm might want to use the same price categories as rival companies so as to have better benchmarks against which to measure relative performance of its managers).

Finally, far from representing a step towards collusion, a competitive firm may want to

select the same price categories as rivals so as to facilitate price comparisons by consumers, and be able to steal more business to rivals through aggressive price behavior (a given price cut will attract more customers when they can easily compare prices than when they have to perform complex calculations).

Indeed, if homogeneous price categories may have a pro-collusive effect because they enhance observability of firms' actions (a crucial ingredient for collusion), they will also facilitate price comparisons by consumers, thereby increasing transparency on the demand side and thus making the market more competitive. There is no robust theoretical result that determines which of the two effects is dominant. However, the empirical evidence we review does show that in markets where prices are more transparent (due to the existence of price advertising, or of internet search engines, or other mechanisms that facilitate price comparisons by consumers) prices indeed tend to be lower. This suggests that the net effect of homogenous price categories might well be pro-competitive.

The results of our study thus indicate that there is not enough ground to suggest that Competition Authorities should devote extra efforts to investigate sectors characterized by homogeneity of price categories (let alone, of course, to conclude that using the same categories should be considered per se unlawful behavior).

Nevertheless, if during an investigation a Competition Authority found that firms had explicitly coordinated to adopt the same price categories, then such an agreement should be considered itself in breach of article 81 of the Treaty (article 2 of the Italian law), even absent evidence of agreements on price levels (or market-sharing arrangements, or output restrictions). Indeed, one can safely presume that if firms explicitly agreed to choose the same price categories, they did so in order to make it easier to sustain collusion (and expected that - for some reasons which might be due to the particular sector in which they operate - transparency on the demand side was unlikely to disrupt collusive outcomes).

In other words, *agreements* to coordinate on price categories should be considered from the legal point of view in the same way as agreements to exchange disaggregate information among competitors: a facilitating practice that is proof of infringement of competition laws.

If, instead, homogeneity of price categories was the result of independent choices, no legal action should be taken: this is because there exist many reasons why firms might want to adopt the same price categories as rivals for purely innocent motives, and because there is no observable element which could unambiguously indicate whether the firms wanted the same price categories to facilitate a collusive outcome as opposed to other reasons.

Contents

1 Introduction, and objectives of the report

There are a number of industries where competing firms choose the *same* (or homogenous) price categories, a *price category* being defined as a situation where a firm commits to sell a given set of products (or to supply a given set of markets), at a single price. In order to clarify the meaning of price categories, let us make some examples.

Airlines. In the industry of airborne civil transportation, different airlines traditionally offer their products under very similar price categories. For instance, the former European incumbent monopolists, such as Alitalia, Lufthansa, Air France, KLM, and Iberia, usually offer their seats under two major price categories, *business* and *leisure* (itself divided into pex, apex, superapex, week-end), whose definitions are similar and depend on the same requirements, such as the possibility to modify the ticket, the number of days spent at destination, whether the trip includes or not spending Sunday at destination, how much in advance the ticket is bought and so on. Furthermore, within a price category there are additional sub-divisions according to the age of the travelers. A casual inspection of the websites of the major European airlines, for instance, shows that most of them use the same price categories: infant (less than two years old), child (from two years to eleven), adult (from twelve years onwards).[1]

Recorded music. Each major record company produces each year a great number of records, which are extremely differentiated products, along several dimensions. Records differ by genre (for instance, classical, jazz, or pop music), by geographical origin (national v. international), and by authors (within the same genre, there are clearly strong differences among artists, and even different records by the same artist might not be close substitutes). Despite this plethora of different products, the major record companies use a very simplified price profile, and - more particularly - they tend to use the same price categories. Indeed, CDs are usually sold by majors in four different price categories: singles, full-price, mid-price, and budget price.[2]

Cinemas. One surprising feature in the price profiles of cinemas is the substantial price uniformity of the tickets sold by the same cinema, independently of the films showed and of the times and days they are showed. Indeed, there exists a substantial variance in the demand for cinema going. Some films are clearly more popular than others, and attendance

[1] This is the case for Alitalia, Iberia, Lufthansa, British Airways, KLM. Some airlines make some further distinctions. Air France, for instance, uses also price categories 12-17, 18-24, 25-59 and over 60 years; TAP differentiates the price category 12-24 from the other adults. All websites visited on 20 July 2004.

[2] See for instance Provvedimento AGCM I207 Associazione Vendomusica/Case discografiche multinazionali-Federazione Italiana Industria Musicale Italiana, 9 October 1997, Bollettino 41/1997.

on Saturdays evening is certainly much higher than attendance in the afternoon of weekdays. Perhaps even more surprisingly, whenever different price categories are introduced, they tend to be homogenous across cinemas. In most towns, for instance, there is a day of the week (e.g., Monday in Barcelona, Wednesday in Florence) where there are special discounts in all the cinemas. That is, all competing firms adopt the same price categories: one low price category for all the movies in that same and certain day of the week, and another, high price category, for all other days.

Car rentals. Car rental companies divide their price offers according to the type of cars demanded by the client. That the price depends on the quality of the car is certainly not surprising. However, the companies rent a large number of types of cars, but they group them in very similar classes. In other words, they use very similar price categories. For instance, Hertz uses the following categories: economy, compact, mid-size, standard, full-size (divided into premium, luxury, 4-wheel drive, minivan, convertible); Avis: subcompact, compact, intermediate, full-size (divided into premium, luxury, sport utility vehicle, minivan, convertible); Europcar: mini, economy, compact, intermediate, standard, full-size (divided into premium, luxury, minivan, special cars).

Telephone services. While mobile telephone services in Italy appear to be characterized by a plethora of very different price profiles and special offers, fixed telephone services have been characterized (at least until the recent introduction of special price profiles such as the flat fee profiles offered by Wind-Infostrada) by the use of very similar price categories. For instance, both Tele2 and Telecom Italia distinguish phone calls among local, national, international, and to mobile operators, with each price category being subdivided further according to the time of the phone call: both operators charge a price for phone calls taking place from 8.00 to 18.30 of weekdays, and another price for phone calls taking place at night or at week-ends, another example of similar price categories.[3] Also the main Spanish telecom operators (Telefonica, Auna and Jazztel) use the same price categories: metropolitan, provincial, interprovincial, fixed to mobile, international. Also, they use the same time schedules. For instance, for metropolitan calls, the high price is set from 8.00 to 20.00 on weekends.[4]

[3] Differences in price categories exist even among these two operators though. For instance, whereas national phone calls are divided in the same price categories, for local calls Telecom charges the high price also on Saturday mornings. Further, international phone calls' price categories differ across countries. While country groups are very similar (lowest prices are assigned for phone calls to EU and North-America, for instance), some differences exist.

[4] Oddly enough, in Spain and in Italy the peak and off-peak divisions are different. Arguably, people's behaviours are different in Italy and in Spain, but not to the extent to justify that off-peak starts at 18.30 in

These examples raise the following question. Why do different firms so often choose to define price categories in the same or very similar way, out of the infinite possibilities available to them? Consider for instance the price categories for age profiles used by airlines. It is normal that airlines want to use different prices according to the age of the travellers (because one can presume that - other things being equal - adults have higher incomes than young people, and because children usually travel accompanied, and it makes sense to offer discounts for families), but the breakdown of price categories according to age is in principle completely arbitrary: to define as an adult a 12 years old and as a child a 11 years old amounts to setting an arbitrary dividing line which probably does not correspond to any standard definition. So, why is it that most airlines instead use the same age price categories?

This Report aims at answering this question, by identifying the possible reasons why firms might want to choose the same price categories (a topic that, to the best of our knowledge, has not been studied in the economic literature). We shall see that behind this homogeneity of choices there will often be innocent behavior, that is, the firms' attempt to maximize their profits in a non-cooperative oligopolistic framework, but collusive motives might also be a possible reason for homogeneity of price categories in some cases.

Building on the positive analysis of the choice of price categories, we shall also make suggestions as to how the issue of price categories homogeneity should be treated the Competition Authorities.

Interestingly, to the best of our knowledge, the issue of homogeneity of price categories has not been under the scrutiny of antitrust authorities so far, or at least has not led to any negative decision.[5]

Plan of the Report The Report is structured in the following way. Section 2 explains why firms might want to adopt price categories at all. To this effect, we first briefly discuss price discrimination, and then list possible reasons - such as transaction, menu and search costs - why firms might want to set price categories (which involves setting a common price for different products/markets, and is therefore a departure from price discrimination) rather than fully price discriminate. For the rest of the Report, we shall take these reasons as given, and focus instead on the reasons why firms (assuming that costs of different nature lead them to adopt price categories) might want to use the *same* price categories.

Italy and at 20.00 in Spain.

[5] We were not aware of any case on this topic ourselves. We have then hired a PhD student in the Law Department of the EUI as research assistant, to identify possible cases, and he has not been able to find any, either in the US or in the EU jurisprudence.

Section 3 argues that whenever firms are symmetric, in the sense that the ranking of their markets or of their production costs is similar, they will tend to select the same price categories. In other words, homogeneity of price categories might quite simply mirror the importance of markets or the efficiency of costs, and whenever their order is symmetric across firms, they will come up with the same price categories through pure non-cooperative behavior. Another - innocent - reason why firms might have the same price categories is due to incentive contracts: for a firm to give the right incentives to its managers, it might be optimal to offer them a compensation which at least partially is based on how well they perform relative to rival companies. Choosing the same price categories might help build better performance benchmarks, and thus more effective incentive schemes.

Section 4.2 wonders whether there might be less innocent reasons why firms might end up choosing the same price categories. The possibility that homogeneity of price categories is a practice that facilitates collusion is explored. Indeed we find that there might be circumstances whether firms might have an incentive to coordinate on the same price categories, as a way to make collusion more likely. Strategic reasons due to committing to the same price categories, the fact that they enhance observability of rivals' actions, and the possibility to better reach an agreement, are arguments that help explain why homogeneity of price categories might help firms both to reach a collusive agreement and to enforce it. However, we shall also argue that - whenever it enhances transparency on the consumer side as well - homogeneity of price categories might well have the opposite effect, inducing consumers to shop around and thus resulting in an increase of market competition. Therefore, it is far from clear that - assuming they want to collude - firms will always want to adopt the same price categories.

Section 5 will summarize the main arguments of the Report, and draw some policy implications. In particular, we anticipate that there is not enough ground for taking actions against price categories. Although it cannot be excluded that homogeneity of price categories might be chosen for collusive reasons in some circumstances, there are too many innocent reasons why firms might want to adopt the same price categories. We shall argue not only that firms (independently of each other) choosing the same price categories is no evidence of unlawful behavior, but also that it is not necessarily a sign that the market should be investigated further. However, if an investigation disclosed that firms have coordinated their actions so as to choose the same price categories, then we should infer that this coordination was due to collusive reasons, and accordingly consider it as an infringement of the law.

To write this Report, we both have built on the insights of several branches of the economic

literature dealing with related issues, and developed our own original formal analyses (as we have already said, homogeneity of price categories has never been the object of economic research). To keep the presentation as simple as possible, however, we keep the Report non-technical, and we relegate the presentation of our formal analyses to the Appendices in Section 6. There, the reader who is interested in the technical details will find a detailed presentation of the models on which this Report is based.

2 Price categories

In this Section, we briefly discuss what is price discrimination, why firms generally have incentives to price discriminate, what is the relationship between the concept of price discrimination and that of product differentiation, and under what circumstances firms might prefer to resort to (a limited number of) price categories rather than fully price discriminate.

We identify three main reasons for choosing price categories, each corresponding with a type of costs: on the supply side, transaction and menu costs; on the demand side, search costs of consumers.

For most of the Report beyond this Section, we shall focus our attention on the reasons why firms choose the *same* price categories, and we shall assume that transaction, menu, or search costs exist, so that they do want to adopt price categories. Nevertheless, it is important to understand why price categories are often chosen by firms at all, and this is precisely the objective of this Section.

2.1 Price discrimination

Price discrimination is a phenomenon that we encounter everywhere. Books are sold at different prices according to whether they are hardback or paperback; journals charge a higher price for libraries and institutions than for individuals (and sometimes make further discounts to students); airlines apply very different tariffs not only for business versus economy trips, but often charge very different prices for the same type of seat and trip according to whether the passenger is a student, a senior citizen, has booked early or at the last minute, and so on.

The objective of price discrimination is simple: firms maximize their profits if they are able to charge each consumer with his/her valuation for the product they sell. Suppose a firm knows that Mr. A values its product 80 and that Mrs. B values it 50. Then the firm attains its maximum possible profits when it price discriminates, and sets the price 80 to A and the price 50 to B. Selling the product at the same price would entail lower profits (at 50, both A and B would buy, but it would get 100 rather than 130; at 80, only A would buy, and profits would be even lower; similarly, for all other prices).

To give a precise economic *definition* of price discrimination is not an easy task. The most natural definition which comes to mind would probably be that price discrimination refers to a situation where the same firm sells *exactly* the same product at different prices. However, Tirole (1988: 134) warns that "[a] general equilibrium theorist might rightly point out that goods delivered at different locations, in different states of nature, or of different quality are distinct economic goods and thus that the scope of 'pure' discrimination is very limited."

In other words, we have to accept that - to be a meaningful concept - price discrimination should also refer to similar but differing products. From this perspective, the best definition is probably the one first proposed by Stigler and then also adopted by Varian (1989: 598): *there exists price discrimination whenever two or similar goods are sold at prices that are in different ratios to marginal costs.*[6]

Different types of price discrimination For price discrimination to arise, a firm must be able to *sort* consumers so as to charge them different prices.[7] Under *first-degree* (or perfect) price discrimination, for instance, a monopolist would know each consumer's precise willingness to pay for its good, and charge each of them exactly the maximum they would pay. Under *third-degree* discrimination, a firm charges different prices to consumers having different (observable) characteristics. For instance, a child might be charged a lower fare for a plane ticket; a citizen over 65 years old might receive discounts on train tickets; the same product might be sold at a different price in two different cities (or regions, or countries). Subject to the qualifications made above (about the extent to which goods sold in different times/places are the same or different goods), here we have the *same product* that is sold to different consumers (whose characteristics can be observed - for instance by requesting to show an ID card) at *different prices.*

Under *second-degree* discrimination, a firm offers different deals to all consumers and lets each of them "self-select", that is choose one particular deal: consumers sort themselves into different groups. Quantity discounts are a natural example: a cinema, for instance, might offer to all customers the option to pay a lower unit price for a film if they buy a ten-films card, but some customers will prefer just to pay one film ticket at a time. Another example is the difference between business and economy tickets: airlines know that firms have a higher willingness to pay than individuals, and they try to make business travellers self-select by offering different tickets which do not require spending Saturday nights away from home and whose time/day of flights are flexible. And again, libraries are able to spend more than private citizens, and since the former tend to prefer hardcover books because they are more resistant, the same title is offered under hardcover and paperback. In all these cases, price

[6] Note that if a firm charges the same price to two different consumers, but it has to pay a higher cost to ship the good to one than to the other, this will effectively be price discrimination.

[7] Another crucial condition for price discrimination to occur is that *arbitrage* must be absent, or that the firm could successfully prevent consumers from reselling the goods among each other (otherwise, those who are charged a lower price would resell to those who should pay a higher price, thus resulting in price uniformity at equilibrium). Arbitrage does not play a big role in determining the choice of price categories, and is accordingly not discussed at length in this Report.

discrimination involves creating differences in the products so as to be able to charge very different prices to consumers having different preferences. Here, *products* are made *different*, in order to sell them at *different prices*.

Price discrimination and product differentiation Apart from identifying the main features of price discrimination, the discussion above helps understand the relationships between price discrimination and product differentiation. In particular, we should stress that price discrimination is not incompatible with the existence of different product characteristics. Indeed, product differentiation might be 'artificially' introduced by firms in order to screen different consumers whose different willingness to pay would not otherwise be exploited. Furthermore, while it is true that we cannot talk of price discrimination when two completely different products are sold at different prices, this concept also refers to similar products that are offered at different prices/marginal costs ratios. Obviously, however, the line between similar and different products might well be a very fuzzy one, leaving room for ambiguity in practice as to whether there is price discrimination or simply the price differences are due to product differentiation.

Note, however, that this issue - however interesting - is more a theoretical one than one bearing practical implications. To go back to the main objective of this Report, we next define price categories, explain how they relate to price discrimination, and why they might be adopted by firms.

2.2 Existence of price categories: why firms might not want to fully price discriminate

As said above, price discrimination is very widespread, and for a very simple reason: the more a firm is able to sort consumers according to their valuations (or willingness to pay) and charge them accordingly, the higher its profits. This sub-section discusses why in some cases firms might prefer not to fully discriminate, and choose price categories instead. Before starting the discussion of why price categories exist, however, it is useful to introduce some definitions which will be used throughout this Report.

Definitions Suppose a firm sells the same product in several different markets (or that it sells several different products). We call a *price category* a situation where the firm commits to sell at the same price in two or more - but not all - of these markets (or two or more - but not all - of its products).

We say that a firm chooses *price uniformity* when it commits to sell at the same price in *all* of these markets (or *all* of its products).[8]

We say that a firm engages in *(full) price discrimination* when it remains free to set a different price in each of the markets in which it sells (or each of its products). Note also that this does not necessarily mean that prices will necessarily be different at equilibrium, but simply that the firm prefers not to commit to sell in different markets (or sell different products) at the same price.

Price categories and different products From the above definitions, it is already clear that the concept of price category might refer to the following two conceptually different situations. First, the *same product* is sold to very different groups of consumers or in different markets, but rather than charging a price for each of these consumer groups/markets the firm is setting a common price for a subset of them. Second, *different products* are sold (in one or several markets), but rather than setting a distinct sales price for each product, a firm is setting a common price for a subset of them.

Examples for the first type include the following cases: airlines, when rather than charging a different price for each age of travellers, they set common prices for given age intervals; fixed telephony companies, when they only roughly differentiate across calling times, charging for instance high prices for weekdays from 8 to 18.30, and low prices at all other times, without making no further distinctions (presumably, around lunch time demand is lower than in the mid-morning).

Examples for the second type include the case of different records sold at the same price; of different movies/same movies showed at different times/days offered at the same ticket and so on.

We now turn to the question of why firms might choose to have price categories rather than fully price discriminate. We identify three main reasons for this, corresponding to three different types of costs, the first two directly faced by the seller, the last faced by the buyers.

2.2.1 Transaction Costs

Broadly defined, the concept of *transaction costs* encompasses all the costs associated with conducting exchanges. Transaction costs may explain why firms sometimes offer price categories. To make this point clear, consider the example of a fruit stall selling oranges. Oranges

[8] Note that price uniformity might also be seen as a degenerate case of price categories, where the firm commits to set a common price in all products/markets.

differ in size, color, juiciness, and several other dimensions. Consider size. Consumers are typically willing to pay more for larger oranges. The stall vendor may want to sort oranges in different grades. It may be reasonable to sort oranges in two, or possibly three grades, but most likely not more because there are costs associated with having many grades. The vendor has to sort oranges, price each grade according to market conditions, and deal with customer complaints in the event of sorting errors. It may even be necessary to re-design the stall to accommodate more grades. In addition, consumers may take time to select the grade they want. They might be turned away if some grades are sold-out and look for their preferred grade elsewhere. Each additional grade imposes transaction costs. This explains why oranges are typically sorted in a few grades only.

More generally, firms may prefer to sell differentiated products at the same price—a price category—rather than pricing the products independently, when there are transaction costs associated with making multiple offerings. This constitutes a potentially important explanation for the observation of standardized categories in many settings.

The transaction cost argument is independent of market structure. In the example above, the fruit vendor could operate in a competitive market. Only a few grades are offered in equilibrium, and those vendors who deviate do not survive. Similarly, monopoly firms may not want to fully price discriminate because there are transaction costs associated with price discrimination. Consider the example of mobile phone carriers. Miravete (2004) observes that monopoly telephone carriers in the early US cellular telephone industry offered only a few tariff options—i.e. a few price categories. This is puzzling in light of the monopoly screening literature, since the standard model of second-degree price discrimination predicts that the optimal non-linear pricing scheme involves a continuum of tariff options. Miravete, however, shows that the incremental profits from an additional tariff decline rapidly as the number of tariff options increases. He points out that even small product development costs, which arguably correspond to the largest component of adding options in that industry, may explain why offering only a few tariff options maximizes profits. Stated differently, price categories save on transaction costs.

2.2.2 Menu costs

A specific kind of transaction costs that may explain the use of price categories is given by the costs that are associated with setting and updating prices. This kind of cost is known as *menu costs* in the macroeconomic literature. The concept of menu cost was introduced to rationalize the existence of price rigidities in the presence of market shocks. In a rare

microeconomic analysis of menu costs, Zbaracki et al. (2004) argue that in addition to the cost of physically updating prices, menu costs involve three types of managerial costs—information gathering, decision-making, and communication costs—and two types of customer costs—communication and negotiation costs. In their case study, they estimate that price adjustment costs comprise 1.22% of the company's revenue and 20.02% of the company's net margin. For the same reason that menu costs may explain why firms may delay price updating, these costs may also explain why firms may sell several products at the same price.

2.2.3 Search Costs

Although most of the explanations presented in this report for the use of price categories focus on supply side arguments, demand side factors may also play a role. To start, consumers may simply have a preference for price categories because it simplifies the process of selecting products. For example, some authors have reported in the context of telephone pricing that consumers have a bias toward subscribing to a flat tariff plan over more complex plans that take into account actual usage (King and van der Ploeg, 1990). For the same reasons, consumers may have a preference for price categories. Confronted with a price category, a consumer only has to find which product s/he prefers and does not have to make surplus computations involving differences between willingness-to-pay and price.[9]

Another demand-side related reason explaining why price categories may emerge is because they may reduce the incentives to search. Consumers incur search costs when they have to choose between many differentiated products which are sold at varying prices. Consider the case of retail shopping. Consumer search costs may arise for several reasons. To start, if consumers expect that prices might vary across stores, but are imperfectly informed about stores' prices, they may want to visit several stores even if it is costly to do so. In addition, consumers may have to visit many stores to find the exact product they want. Finally, consumers may not know which product they like and they may benefit from sampling several products at different outlets.

Competitive pricing with search costs can give rise to equilibrium outcomes that would not occur under perfect competition. For example, Varian (1980) shows that a distribution of prices could arise in equilibrium. In a different model, Diamond (1971) shows that competing firms may be able to sustain monopoly prices even when there are arbitrarily small search costs.

In Appendix Section 6.4, we present a simple model with search costs, in which price

[9] Note that these arguments are based on some form of bounded rationality of consumers.

categories may endogenously emerge in equilibrium. In addition, the model pins down an important interaction between consumer search, the use of price categories, and price competition: namely, firms use price categories to modify consumer search incentives, with the ultimate goal of softening price competition.

In particular, we consider a model in which firms sell products that have different production costs and for which consumers have different willingness to pay. We show that price categories arise when some consumers are imperfectly informed about their preferences and others are perfectly informed. Those who are imperfectly informed may benefit from search because doing so allows them to find a better match.[10] For this reason, price categories are offered in equilibrium to eliminate the uninformed consumers' incentives to search. We recognize that the analysis is sensitive to the assumptions of the model. Therefore, the model should be interpreted with care.

2.2.4 Softening competition

It is also conceivable that firms might want to adopt price categories, rather than fully price discriminate, in order to make competition softer (even in the absence of search costs). One might think that by committing to set a single price for several markets might make competition softer relative to a situation where a distinct price can be chosen in each market. We explore this possibility in the technical Section 6.3 of the Appendix, and find that for some particular market configurations, firms may indeed use price categories as an alternative collusive device when they cannot directly collude on prices. More precisely, we show that firms might include in a price category only markets that differ according to their level of competition. Section 6.3 provides a detailed analysis and a discussion of this case.

Conclusions This Section has reported some of the reasons why firms might want to commit to sell different products (or to different types of consumers) at the same price, instead of price discriminating. We have argued that transaction costs (and a particular form of them, menu costs) and search costs might induce firms to simplify their price profiles, and prefer price categories to full price discrimination.

For the remainder of this Report, we take for granted that such costs exist, and focus instead of the question why - if firms use price categories - they want to adopt the same categories, rather than different ones.

[10] As in Diamond (1971), we show that firms perfectly price discriminate when there are no imperfectly informed consumers.

3 Which price categories? Non-collusive motives behind the choice of identical price categories

In this Section, we investigate whether there exist non-collusive motives that might lead different firms to choose the same price categories (as explained above, we assume that transaction, menu or search costs exist, so as to justify the use of price categories). We first (Section 3.1) describe the result of our own model, in which firms non-cooperatively decide on which price categories to commit (for a given number of price categories), before choosing prices. We use it to explain why symmetry among firms (in demands, costs, or market sizes) will lead them to choose the same price categories. Then (in Section 3.2.1), we turn to a discussion of the recent literature on principal-agent models, and we argue that another innocent reason why firms might choose the same price categories so as to construct performance benchmarks and to provide their workers with more effective incentive schemes.

3.1 Symmetry among firms lead to identical price categories

To the best of our knowledge, while there is a huge economic literature on some related issues, such as price discrimination or product differentiation (or rather, why firms might want to choose the same products), the question of whether firms might want to choose the same price categories has not been the object of either formal or informal investigations in economics.

To deal with this issue, we have resorted to our own original analyses based on modern oligopoly theory. Let us briefly discuss our setting and report our results informally here (the reader can refer, for the full analysis of the model, to the Appendix, Section 6.1).

Suppose that in a given sector there exist two rival firms, A and B, and that each of these firms sells in three different product markets (or sell the same product to three different types of consumers), 1, 2, and 3. (The same arguments apply for a larger number of firms and markets, but reducing these numbers to the minimum necessary helps focus on the main insights without being lost by the complexity of the different possibilities available.)

For simplicity, assume that there is no possibility of substitution across markets, whereas there might be substitutability between the products sold by the two firms in each market (i.e., there might be product differentiation between firms' products).

One could think of different reasons why a given firm might regard the three markets as different, such as the consumers' valuation across markets, market sizes, the cost of producing or selling in each market, or the degree of substitutability between products in each market. For instance - other things, including the choices of its rival, being equal - firm A might know

that consumers in market 1 have a higher valuation for its product than consumers in market 2, who in turn have a higher valuation than consumers in market 3. It would be equivalent to state that the cost of producing for consumers in market 1 is the lowest, followed by the cost of producing for market 2, with the cost of producing for market 3 being the highest.

We know from the basic principles of price discrimination that in these circumstances a firm would like to charge different prices in the three different markets. To continue the example above, for instance, firm A would want to price discriminate, and - given their decreasing order of marginal profitability - choose the highest price in market 1, followed by a lower price in market 2, and finally by the price in market 3, which will be the lowest. Indeed, it is easy to show that - independently of what the rival does, full price discrimination would be the dominant strategy for each of the firms.

But suppose now that full price discrimination is not profitable, for instance because it would involve too high menu costs, so that each firm has to set at most two prices. We show in Section 6.1 that each firm will tend to set a common price in the two markets which look 'more alike' each other. Suppose for instance that the markets are identical in all other dimensions, and that consumers have identical valuations for B's products, while consumers' valuation for A's products follows the example above, with the additional hypothesis that the valuation of market 2's consumers for A is closer to that of market 1's than of market 3's consumers (in technical terms, $v_{2A} > (v_{1A} + v_{3A})/2$). It can be proved that firm A finds it optimal to put markets 1 and 2 in the same price category, and to leave the price of market 3 independent. The intuition for this result is simple: the first best for the firm would be to choose different prices for each market, and having to choose the same price for a pair of them entails a loss in its profits. Such a loss would be the higher the more dissimilar the markets from each other. Hence, the second best will be to find which of the two markets are more similar, and set a common price in the two of them. This insight can be best understood by thinking of the extreme situation where two markets, say 1 and 2, present identical valuations: in this case, choosing to price them in the same way would not entail any foregone profits.

This insight can now be used to move further, and to think of what happens when both firms have to choose price categories. Sections 6.1.4 and 6.1.5 consider the full game where each firm first has to commit to one price category and then, given the price configurations chosen, firms have to decide on the price levels at which they want to sell. In Section 6.1.4, that we call the *symmetric* case, we assume that firms rank markets in the same way and that, for each firm, markets 1 and 2's valuations are more similar to each other than for any

other pair. Under these assumptions, firms will choose to set the *same* price categories, that is, each of them will set a common price in the markets 1 and 2. By denoting in brackets the markets which belong to the same price category, and doing this for each firm, this equilibrium outcome can be labelled as (12,12).

In Section 6.1.5, instead, that we call the *asymmetric* case, we assume that firms have mirroring market valuations for their products: firm A ranks 1, 2 and 3 in decreasing order, with 1 and 2 being the 'similar' markets in the sense above mentioned, while firm B ranks 3, 2, and 1 in decreasing order, and considers 3 and 2 as the 'similar' markets.

We find that the result of the game, which is purely non-cooperative, is completely driven by the firms' ranking of markets. In the former (symmetric) case, at the equilibrium both firms choose the same price categories, whereas in the latter (asymmetric) case, at the equilibrium firms end up choosing different price categories. More precisely firm A will choose to sell 1 and 2 at the same price category, and firm B will sell 3 and 2 at the same price category. In short, this equilibrium can be denoted as (12,23).

However simple, this model sheds light on an important effect: even when strategically interacting, oligopolistic firms might be induced to select the same price categories for innocent reasons, that is, because they face products or markets that they rank in a similar way. Put it another way, choosing the same price categories might well be the result of normal competition in the industry and of the fact that rivals face similar demand or cost conditions.

There will be many situations where firms will find themselves in these circumstances. Fixed telephone companies in the same country will probably find that - although their consumers have somehow different characteristics - they face a much more inelastic demand during working hours of weekdays, when many people call for business related reasons - than in the evenings or at week-ends. Cinemas in a given town will probably experience a fall of spectators' numbers in given weekdays, and might want to set lower price tickets in one or more of them; presumably, restaurants will find that at lunch people will be willing to spend less for their meals, because they have less time during their work breaks than when they go out in the evening for dinner for social reasons, and because at lunch their primary objective is to feed themselves rather than enjoying the company and the food; accordingly, it is not surprising that they might come up with different price deals, one for lunches and the other for dinners.

True enough, these arguments can explain why firms use price categories that are similar (think of the car-rental examples, where car-rental companies adopt categories which resemble each other without perfectly coinciding; or of the international phone calls example, where

fixed line operators group countries in a similar, albeit not identical, way). However, they do not necessarily explain why two firms might end up choosing *exactly* the same price categories. In other words, how should we interpret those cases - admittedly few - where categories are perfectly coinciding?

Apart from the example of restaurants, for instance, where the distinction between lunch and dinner menus might be natural and obvious, leading to two well defined price categories (special lunch menu offers, and regular à la carte prices for dinner), one might wonder why two telephone companies might define the working hours in exactly the same way: why would both firms choose the peak time price category as from 8.00 to 18.30, for instance, rather than from 7.45 or 8.15 to 18.45 or 19.15?

3.2 Other factors that lead to identical price categories

There might exist several reasons why firms might adopt - among all the arbitrary dividing lines that define price categories - exactly the same. A first reason might be *history and habits*. Suppose for instance that an incumbent has already been operating in the sector as a monopolist, and a new firm enters. The newcomer might use exactly the same price categories not to confuse potential users (a senior citizen might be put off by a different definition of price categories that does not grant her the same discount rates; a retailer might have to change his own price profiles if he switched to the newcomer); or simply because some of the newcomer's employees used to work for the incumbent and are used to work with those price categories; or because the management uses the same software already developed for the incumbent (think of airlines, which use a rather sophisticated software to change price levels according to demand; a new airline might be tempted to use the same software bought by a former incumbent, and organized around the same price categories, rather than incurring own development costs or taking risks with a new software).

A second reason might be *strategic*. Think for instance of a newcomer who wants to signal to consumers that it is going to pursue an aggressive strategy, and offer better deals than the incumbent. If sending such a signal is what it wants, the best strategy is to choose exactly the same price categories as the rival, and systematically cut prices in each category. Of course, the opposite is true when the new entrant wants to avoid head-to-head competition with the rival, for instance because the latter is much stronger in terms of assets and financial resources and the risk of a price war is too high. When Omnitel entered the Italian market for cellular phone services, for instance, it was careful to introduce new types of contracts and not to choose the same price categories as TIM, the incumbent, which had already developed

a solid customer basis and was part of the giant Telecom Italia group. Omnitel had to find the way to signal to consumers that it offered better deals than TIM (due to network effects, the latter had already a strong advantage), but at the same time wanted to avoid to make it too apparent, fearing that TIM could have retaliated by engaging a price war that the new entrant could simply not afford.

Incidentally, this is a point that needs to be stressed: when firms adopt the same price categories - provided that they are publicly announced, and not just visible for sellers but not for buyers - transparency on the demand side will usually result in consumers shopping around for the best deals: this stimulates competition, and will result in lower prices, as we shall discuss in Section 4.4. (Admittedly, however, as we shall argue in Section 4, transparency might also make collusion more likely, so these two effects will have to be traded off.)

Finally, another reason why firms might choose exactly the same price categories might reside in more effective incentive schemes, as we discuss next.

3.2.1 Homogeneity of price categories and managerial relative performance evaluation

We briefly discuss another possible rationale for the observation that firms in the same industry sometimes offer identical product categories. The basic idea is that product categories may help firms to design more efficient incentive contracts.

To explain why this may be the case, it is necessary to first briefly review the standard principal agent model of incentive provision under moral hazard,[11] which clarifies the role of "pay based on performance", i.e. of compensations (such as managerial compensations) composed by a fixed wage and by a variable component based on some measure of performance (output, sales, profits, revenues, etc.).

In the standard moral hazard model, a risk-neutral principal hires a risk-averse agent to supply effort. The principal does not observe the agent's effort directly but observes a performance outcome that is a noisy measure of the agent's effort. If the principal does not make the agent's compensation contingent on performance, the agent supplies no effort. If the principal offers powerful incentives (i.e. sells the firm to the agent and makes her compensation fully based on performance) the risk-averse agent faces too much compensation risk. The optimal compensation, which is *partially* based on performance, balances the goals of insurance and incentive provision. Noisy performance measures impose a cost on the principal (and an efficiency loss) because the agent faces residual risk in equilibrium.

[11] See Mirrlees (1974, 1979), Holmstrom (1979) and Shavell (1979).

The higher the variance of performance noise, the less important the role of the variable component.

However, shocks to individual outcome may have an *idiosyncratic* component which is specific to the agent and an *aggregate common* component which affects also other agents (shocks to market demand, shocks to input availability or costs, etc.). In these instances, as shown by Holmstrom (1979) the optimal incentive scheme should try to filter out common risk from individual compensation contracts, since aggregate shocks make outcome a *noisier* signal of the agent's effort. Removing the common shocks from the indicator on which the variable compensation is based allows to construct a less noisy signal of the agents' effort, thereby increasing the importance of the variable component in the overall managerial compensation, and providing stronger incentives.

A natural way to obtain information on the common aggregate shocks is to look at the performance of other comparable agents, like managers working on similar tasks in the same unit or in rival firms. This filters out the common shocks but subjects the manager to the risk of idiosyncratic shocks experienced by others. Holmstrom (1982) shows that, if idiosyncratic shocks are normally distributed with zero mean, the optimal managerial compensation must condition not only on individual performance but also on the average performance of comparable agents, i.e. the variable component must be based on *relative performance*.[12]

Performance benchmarking may explain why firms use the same price categories. Assume that firms face menu costs and must use some price categories. A firm may benefit from using the same price categories as those used by other firms. To keep the argument simple, assume that each price category is under the control of a single agent. It could be a single manager or a team of managers. Including the same markets/products as rivals in a given price category implies that managers' performances for that category are subject to the same aggregate shocks. Therefore, a firm using the average industry performance for that price category will have a good benchmark for her own agent's performance. Alternatively, if the firm would decide to pool markets/products in different price categories than its competitors, it would be more difficult to construct performance benchmarks. *This suggests that a firm might decide to choose price categories similar to the ones chosen by rivals in order to be able to design more effective incentive schemes.*

[12] See Gibbons and Murphy (1990) for evidence on managerial contracts with this feature.

Conclusions This Section has identified several reasons why firms might end up choosing the same price categories through purely non-cooperative behavior. Firms' similarities in demand or supply conditions, attrition due to history and habits, effectiveness of incentive schemes, or even the desire to appear more competitive than rivals, might all explain the adoption of identical categories as a normal competitive phenomenon.

We now turn to possible anti-competitive reasons for the adoption of identical price categories.

4 Which price categories? Possible collusive motives behind the choice of identical price categories

In this Section, we analyze the possibility that firms might choose price categories in order to sustain collusion more easily. The Section is organized as follows. Section 4.1 briefly defines collusion and sets the stage for the analysis. Section 4.2 describes the main results of our own analysis (while the description of the formal model it is based upon is relegated to the technical Appendix 6.2). Section 4.3 builds on previous work to extend and discuss the analysis on the possible collusive effects of homogeneity of price categories, which might help firms solving coordination problems and might increase market transparency on the producers' side. However, homogeneous price categories might also improve market transparency for consumers, as discussed in Section 4.4. To avoid this, firms might prefer to choose different price categories. Section 4.5 makes some summarizing and conclusive remarks on the topic of collusion and price categories.

4.1 What is collusion?

Collusion is a situation where firms' prices are higher than some competitive benchmark,[13] or close enough to the prices firms would set if they could set monopoly prices (that is, if all of them belonged to the same group). It is not easy for firms to achieve a collusive outcome, even if they are free to agree on the prices they set. In particular, every firm would have the temptation to unilaterally deviate from a collusive scheme, as by doing so it would increase its profit.

The acknowledgment that any collusive situation naturally brings with it the temptation to *deviate* from it and therefore to break collusion, leads to the identification of the two elements which must exist for collusion to arise. First, its participants must be able to *detect* in a timely way that a deviation (a firm setting a lower price or producing a higher output than the collusive levels agreed upon) has occurred.[14] Second, identifying the deviation is not enough: there must also be a *punishment*, which might take the form of rivals producing much higher quantities (or selling at much lower prices) in the periods after the deviation,

[13] For instance, in a homogenous goods game where firms choose prices, a collusive outcome would exist whenever prices are higher than the one-shot Bertrand equilibrium price; where firms choose quantities, whenever they are lower than the one-shot Cournot equilibrium quantities.

[14] In many markets, firms' prices and outputs are not directly observable, for instance because list prices might differ from transaction prices, or because list prices do not even exist, but are set by negotiations between buyers and sellers.

thus depressing the profit of the deviator.[15]

Only if a firm knows both that a deviation will be identified quickly and that it will be punished harshly (i.e., it will have to forego enough profits because of the market reaction of the cartel members), might it refrain from deviating, so that the collusive outcome will arise.[16] A firm contemplating a deviation will expect that the rival will retaliate with an aggressive pricing policy after a deviation. As a result, the gains made by a deviation today must be traded off with the prospect of a price war, and therefore lower gains tomorrow.

These insights are at the basis of the analysis of collusion in modern industrial economics. For collusion to arise, each firm must have its *incentive compatibility constraint* for collusion satisfied. Each firm compares the immediate gain it makes from a deviation with the profit it gives up in the future, when rivals react. Only if the former is lower than the latter will the firm choose the collusive strategy. In general, collusion is more likely to arise the lower the profit that a firm would obtain from deviating, the lower the expected profits it would make once the punishment starts, the more weight firms attach to the future (i.e., when the "loss from deviation" occurs).

To understand how selecting the same price categories might help collusion, we use this conceptual framework, and see how homogeneity of price categories would affect the firms' incentive constraints for collusion. In what follows, we describe our own analysis (which is fully formalized and explained in Section 6.2 of the technical Appendix).

4.2 Non-technical description of the results obtained in the dynamic section

In Section 3 above, we have seen that if firms play a game where they first decide on price categories and then compete in prices only once, the equilibrium outcome will depend on demand (or cost) characteristics. In particular, firms will choose similar price categories if they face similar valuations for their products, and will choose different price categories if consumers' valuations are asymmetric.

The next question to ask is the following. Suppose we observe that in a given industry

[15] Generally, a punishment also hits the punishing firms, and not just the deviating firm, because it has to rely on market mechanisms, typically lower prices or larger quantities brought on the markets (which negatively affect all the firms' profits).

[16] In turn, this implies that collusion can be sustained only if firms meet repeatedly in the marketplace. Otherwise, a punishment cannot take place. In technical terms, collusion will never arise in a one-shot game. This is why collusion should be modelled through dynamic (repeated) games, as we do in the Appendix, Section 6.2.

different firms choose identical price categories. Can we infer that this is a natural outcome of a situation where firms are competing, or might it also be due to some sort of anti-competitive behavior? Clearly, the former interpretation is possible, as we know that it might be the result of normal competitive interactions, bacause firms are symmetric or for other reasons discussed in Section 3. In order to explore whether the latter may also hold, let us consider the asymmetric case and see whether firms might have an incentive to coordinate on the same price categories to sustain collusion, rather than choosing different price categories.

To this end, our model, which is fully described and analyzed in technical Section 6.2, compares the likelihood of collusion in two alternative configurations: the first, denoted as (12,23), where firms use the (different) price categories that arise as the equilibrium outcome of the game analyzed in Section 6.1.5 and summarized in Section 3; the second, denoted as (13,13), where firms choose the same price categories, and more particularly set a common price in their strongest and weakest market, letting the intermediate market price be independent. (Recall that in the asymmetric case the strongest market for one firm corresponds to the weakest market for the rival, and vice versa.)

In order to identify the likelihood of collusion associated with these two configurations, we assume that the price categories choice made by the firms at the outset of the game cannot be changed for the whole game, and that after the commitment on categories is taken, firms will repeatedly interact in the marketplace for an infinite number of periods. As explained above, the likelihood of collusion is determined by how slack or tight the incentive constraint for collusion is in each of the two configurations.

Our analysis shows that choosing the same price categories (13,13) will unambiguously facilitate collusion, for the following reasons.

First, the *collusive profits* are larger when the firms adopt this configuration. Indeed, the highest collusive profits that firms can attain are those they would have if they colluded and fully price discriminated. In this case, the highest joint profits would entail setting the prices at which each firm acts as a monopolist in the strongest market (where the consumer valuation is highest) and sharing demand at the joint profit maximisation prices in the intermediate market. In other words, firm A would set the monopoly price in market 1, and be the only seller there; firm B would do the same in market 3; and both firms would set the monopoly price in market 2 and equally divide demand (this does not require a market sharing agreement, consumers would be indifferent between the two firms in market 2, and each firm would approximately get half of the market).

It turns out that under the price category configuration (13,13) firms can exactly replicate

this outcome. Each firm would sell in both the strongest and the weakest market at the price which corresponds to the monopoly price in the strongest market, and in the intermediate market at the price which maximizes joint profits. At this collusive equilibrium, the same would happen as under the collusive outcome of price discrimination: firm A would achieve monopoly profits in market 1 and obtain no sales in market 3, and vice versa for firm B. The highest collusive outcome can therefore be implemented.

Under the configuration (12,23), instead, the collusive outcome of price discrimination cannot be replicated: firm A must offer the same price in markets 1 and 2, and firm B the same price in markets 2 and 3, while the first best for the firms would be to have different prices in these two markets (the two can coincide only in the degenerated case where market valuations in 1 and 2 are the same for firm A, and market valuations in 2 and 3 are the same for firm B).

Since the higher the collusive profits, the more likely that the incentive constraint for collusion is satisfied, this first effect makes collusion more likely under the homogenous price categories configuration (13,13).

Second, consider the *deviation profits*, that is the maximum profits that a firm that deviated from the collusive outcome would achieve. Under the configuration (13,13), each firm is obliged to set the same price in its strongest and weakest market. Consider firm A (the same reasoning would apply to B, mutatis mutandis): given that the rival sticks to the collusive strategy, firm A can increase its profits by slashing prices and stealing the rival's business in markets 2 and 3. But stealing market shares in market 3 is very costly for it, since it would entail reducing the market price in its market 1, where it has a monopoly at the collusive equilibrium. Further, given that firm A faces lower valuations in market 3, it would have to cut prices considerably to be able to capture market shares in market 3.

Under the configuration (12,23), instead, firm A can be comparatively more aggressive when it wants to deviate. Indeed, it is free to reduce prices in market 3 as much as it wants; this time, it is cutting prices in market 2 that is a costly strategy in terms of foregone gains in market 1, but since its valuation in market 2 is closer to that in market 1, stealing a given proportion of business in market 2 requires a less aggressive price cut than the one that needed to steal business in market 3. Therefore, deviation profits will be higher under configuration (12,23).

Since the higher the deviation profits, the *less* likely that the incentive constraint for collusion is satisfied, this second effect again makes collusion more likely under the homogenous price categories configuration (13,13).

Finally, consider *punishment profits*, that is the profits that firms would obtain after a deviation has taken place and a price war has thus been triggered. For simplicity, we assume in our model Nash-reversal strategies, that is, we assume that along the punishment path the firms would fall back to the Nash equilibrium of the one-shot price game. It is then immediate, from our analysis of the static case of Section 3, that punishment profits are higher under the configuration (12,23) than under the configuration (13,13). Indeed, we know that one-shot profits are the higher the more similar the two markets in which a common price is set. Under (13,13), it is the strongest and the weakest market of each firm that have the same price, thus determining a bigger distortion than under the price category (12,23), where the same price is set for the two most similar markets.

Since the higher the punishment profits, the *less* likely that the incentive constraint for collusion is satisfied, this last effect also makes collusion more likely under the homogenous price categories configuration (13,13).

We can therefore conclude that it is conceivable that asymmetric firms might nonetheless try to coordinate on the same price categories in order to facilitate collusion.

Note, however, that this result should be qualified. First, in our example it is only under one particular configuration of identical price categories that collusion is facilitated: if firms chose identical categories (12,12) or (23,23), collusion would not necessarily be more likely than under (12,23). Second, we should stress that for this result to hold price categories must represent a strong commitment for firms, which is not a weak assumption. In our model, if firms could change price categories after a deviation, then price categories would have no effect whatsoever on the sustainability of collusion.

4.3 Homogeneity of price categories and price transparency (on the producers' side)

In this Section we discuss further reasons why the choice of similar price categories might favor collusion.

First, if firms include the same markets/products in a given price category, it may be easier to identify the fully collusive price for that category, i.e. the price that maximizes their joint profits, and it may be more natural for firms to coordinate on it. (Recall that infinitely repeated games typically have multiple equilibria and coordinating on the most profitable one is not obvious, in particular when firms cannot explicitly communicate with each other, since using the market to signal intentions to coordinate on a specific price might be very costly.) Instead, if a firm decides to pool markets/products in different price categories than

its rivals, it may be more difficult to find out the most profitable pricing policy and to signal to rivals the intention to coordinate on it.

Second, homogeneous price categories might favor collusion through the firms' possibility of better monitoring each other's prices. As illustrated by Section 4.1, detection of deviations is a crucial ingredient for collusion: timely detection implies immediate punishment, and thus increases the firms' incentives to collude.[17] Hence firms have incentives to adopt practices that improve observability of prices and quantities and help identifying whether a deviation has occurred. Homogeneous price categories may play this role.

Suppose that each firm sells dozens of products, or sells the same product in a large number of different markets (or to a large number of different types of consumer). We know that - at least if some conditions are fulfilled - a firm would like to price discriminate in order to maximize its profits, and this would result in each firm setting dozens of different prices, one for each product/type of customer. The existence of such a large number of prices for each firm would make it difficult for the firms in the industry to monitor each other's behavior. At each period, idiosyncratic shocks (each product/market might experience a fall/rise in demand, or an increase/decrease in the cost of the inputs, or the entry/exit of competitors in that particular product/market) as well as attempts to gain market shares to the expense of rivals might result in a high variability of prices in the different products/markets, some increasing, some others decreasing, and possibly in very different proportions. As we explain below, this implies that observing each other's behavior, and in particular detecting deviations, is very difficult. The use of price categories might well improve observability of firms' actions, and therefore favor collusion. Rather than having a number of prices (say, dozens for each rival) to look at, adoption of *price categories* (and, in the limit, of price uniformity *tout court*) will make the task of monitoring rivals much easier, as only few (in the limit one) prices for each rival should be observed. Choosing the *same* price categories *might* simplify this task further: if each price category contains a different set of products/markets, it might be more difficult for firms to understand to what extent a price decrease might reflect shocks or is due to a deviation than if firms chose different price categories.

Although there is no formalization of the idea that homogenous price categories would facilitate collusion (and, admittedly, we have been unable to model it), the economic literature has not failed to indicate a number of other practices that firms might adopt in order to

[17] Stigler (1964) argued that collusive agreements would break down because of *secret* price cuts. In fact, Green and Porter (1984) showed that if actual prices (or price discounts) are not observable, collusion would be more difficult to sustain, but it could still arise at equilibrium, although it would be characterised by periods of high (collusive) prices alternating with periods of low prices (price wars).

facilitate reciprocal monitoring.

For instance, firms can exchange information on past or current prices and quantities. Information on prices and quantities may concern *each individual firm,* thereby allowing to identify deviators and better target market punishments. In the absence of disaggregate information on past prices and quantities, availability of more precise estimates of *aggregate (market) demand* would also help, as it allows firms to see whether a decrease in individual demand is due to cheating of rivals or to a negative shock in market demand.[18,19]

Also Resale (or retail) Price Maintenance (RPM) - a vertical agreement whereby a manufacturer imposes upon its retailer(s) the price at which the good should be sold in the final market - may favor collusion. The intuition is that if the retailer was free to choose the final price, the price would be subject to fluctuations because it would reflect variation over time in the costs of retailing (observed only by local distributors). If wholesale prices are not easily observed by cartel members, cartel stability would suffer because changes in retail prices caused by costs changes would be difficult to distinguish from cheating on the cartel. RPM can enhance cartel stability because final prices fixed by manufacturers would not adjust to retail cost shocks, thereby eliminating the retail price variation and allowing to better identify deviations from a collusive actions.[20]

Finally, when producers are located in different geographic areas, and serve consumers that are also spread out over the territory, it might be difficult for firms to compare prices and to detect price changes, since prices vary with transportation costs. To facilitate price observability among rivals, firms can set uniform delivered prices, i.e. a firm sets the same price inclusive of transportation cost throughout its territory, and independent of the customers' locations. A similar effect is achieved by basing point pricing, a system whereby each producer sets the final price as the mill price at the common basing point plus transport cost from that point to the final destination. Again, this allows to increase transparency on the producers' side, in that it allows to better compare prices.

[18] On collusion and exchange of information between competitors, see Kühn (2001).

[19] Porter (1983) shows that exchange of private information about market demand reduces demand uncertainty and allows more collusive outcomes to be sustained. In a similar vein, Kandori (1992) shows that as demand uncertainty decreases, firms can attain higher collusive outcomes (and punishment phases become more severe), and Kandori and Matsushima (1998) also find that communicating information about past realisations helps collusion. Other papers that deal with collusion under imperfect monitoring and private signals are Compte (1998) and Athey and Bagwell (2001).

[20] This story has been recently formalised by Jullien and Rey (2001).

4.4 Homogeneity of price categories and market transparency (on the consumer side)

As we have just discussed, homogenous price categories might increase market transparency *for producers* and thus favor collusion. However, the choice of homogenous price categories makes it easier *for consumers* to compare prices charged by different firms, thereby reducing search costs and increasing market transparency on the consumer side. This is likely to increase the demand elasticity faced by firms, to make markets more competitive, and in a *static setting* to reduce equilibrium prices. In a *dynamic setting*, it might also make collusion more difficult to sustain, because the increased elasticity of demand enhances the incentives to undercut rivals.

If these effects dominates, firms may well decide to choose different price categories, in order to introduce some friction into the market.

Moreover, this argument points out that whenever homogeneous price categories arise (for instance, in a static model), there might be a welfare-improving effect due to the reduction of search costs.

We provide indirect support for this argument by resorting to the theoretical literature (Section 4.4.1) and to the empirical works (Section 4.4.2) which study the impact of increased market transparency from the consumer side.

4.4.1 Theoretical support

Search models. This literature emphasizes that, even though identical firms sell an homogeneous good, the fact that consumers are not perfectly informed about prices and that the acquisition of information is costly causes equilibrium prices to move away from marginal costs. In particular, two main results are possible. The first is that the existence of arbitrarily small search costs makes equilibrium prices settle down at the pure monopoly price with each firm acting as a complete monopolist over its usual consumers[21]. The second is that there may be permanent price dispersion in the range between the perfectly competitive and the monopolistically competitive price (with firms having a pure strategy and charging different prices or drawing prices randomly from an equilibrium distribution).[22]

In these models, increased market transparency typically exerts a pro - competitive ef-

[21] See Diamond (1971) and Salop and Stiglitz (1977).

[22] Works in this area include Salop and Stiglitz (1977), Reinganum (1979), Wilde and Schwartz (1979), Varian (1980), Stahl (1989), Carlson and McAfee (1991). Price dispersion typically results from either consumers' heterogeneity in their search technology or producers' heterogeneity.

fect. For instance Stahl (1989) studies the effect of an increase in the share of customers with complete information about prices, which is found to shift downward monotonically the equilibrium price distribution. When the share of fully informed consumers is equal to 1, the distribution is degenerate at the competitive price. A similar effect is caused by price advertising, which makes consumers aware of attractive price offers at other locations, thereby increasing the share of fully informed consumers, enhancing competition, and inducing lower prices and higher consumer welfare (see for instance, Robert and Stahl, 1993; Bester and Petrakis, 1995).

These results may be interpreted in the sense that *more similar price categories, by facilitating price comparisons for consumers, increase the share of fully informed consumers, and may therefore cause average prices to fall and consumers to be better off.*

Consumer side transparency and collusion There has been very little work on the effects of consumer side transparency on collusion. An exception is Schultz (forthcoming) who shows that improved transparency, by increasing demand elasticity of a firm, exerts two opposite effects on the sustainability of collusion. On the one hand, the increased elasticity of demand enhances the incentives to undercut rivals. On the other hand, a more severe punishment is possible in a market where firms' demands are more elastic. The paper shows that the first effect dominates when goods are differentiated, so that *improved transparency on the consumer side makes collusion harder to sustain.* In almost homogeneous markets, the effect vanishes and changes in transparency have almost no effect on collusion.

4.4.2 Empirical support

There exist substantial empirical evidence showing that the reduction of search costs and the increase of information available to consumers reduces equilibrium prices. A number of empirical works support this view showing that price advertising causes substantial price declines. Other works provide evidence that the Internet, by facilitating price comparisons on line and thus reducing frictions, makes demand more sensitive to prices and markets more competitive.

Impact of price advertising on pricing behavior Benham (1972), Benham and Benham (1975) and Kwoka (1984) study the impact of advertising restrictions on the price of eyeglasses and eye examinations in the US. Their approach is to compare prices in jurisdictions that forbid advertising (or where professional exert a strong control over the types and quantity of information that can be transmitted to consumers) to prices in jurisdictions that

allow advertising (or where professional control is weaker). *They find higher prices for eyeglasses (or eye examinations) to be associated with restrictions on advertising and professional control over information disclosure.*

Other works exploit the change in the advertising regime occurring at some date. Glazer (1981) identifies the effect of advertising using an exogenous variation in advertising provided by a newspaper strike. Love et al. (1992) focus on the market for legal services in England and Wales where in 1985 for the first time solicitors were allowed to advertise. Milyo and Waldfogel (1999) study the impact of the 44 Liquormart Decision where in 1996 the US Supreme Court overturned a Rhode Island ban on advertising the prices of alcoholic and beverages. *All the papers find price advertising reduces prices charged in the relevant market.* For instance, Milyo and Waldfogel (1999) find that stores running ads reduce the advertised products' prices by about 20%.

Impact of Internet on pricing behavior Internet technologies may significantly reduce search costs by enabling price comparisons on-line. Prices charged by competing retailers can be easily found on the firms' websites. Price search engines, which indicate instantaneously and without charge a sorted list of the prices charges by dozens of e-retailers, strongly enhance price comparability. Thus, the ability of the Internet to reduce search cost may have a significant impact on market power and may lead to large consumer welfare gains.

Brown and Goolsbee (2002) and Scott Morton, Zettelmeyer and Silva-Risso (2001, 2004) investigate the effect of the rise of Internet use on the price of products which continue to be sold off-line. The former paper focuses on the market for term life insurance. The results indicate that once on-line insurance sites began (which provide a wide range of information, including market conditions and pricing policies), *the rise of Internet use from 1995 to 1997 appears to have made the market significantly more competitive and have reduced term life prices by about 8-15%.* The latter papers focus on the car industry. The authors show that, conditional on the car, consumers that contact dealers through Internet referral sites pay on average 2% less than off-line customers (which corresponds to about $450 for the average car). Still, just because Internet consumers are paying less than off-line consumers does not mean that they are paying less than they would if the Internet did not exist. The authors control for potential selection effects and *buying a car through Autobytel.com* (one of the main referral sites) *is shown to reduce the price a consumer pays by approximately 2.2%.* This suggests that consumers choose to use Internet because they know that they would do poorly in the traditional channel, perhaps because they have a high personal cost to collecting information and bargaining.

Ellison and Ellison (2004) and Goolsbee and Chevalier (2003) have analyzed the impact of Internet on demand price-sensitivity. The former paper examines competition between a group of Internet retailers that operate in a segment (computer parts sold by small firms) where a price search engine (Pricewatch) plays a dominant role. The latter paper examines sales of on-line books and estimates the own- and cross-price elasticities of demand at Amazon and Barnes and Noble.com. *Their results show that the easy price search reduces frictions and makes the demand faced by a seller very elastic to the own price.*

4.5 Summary and conclusion on collusion

This Section shows that firms may choose homogeneous price categories in order to facilitate collusion. The main result of our analysis is that committing to the same price categories favors collusion because it allows firms to achieve the highest collusive profits, it makes deviation less profitable and the punishment harsher. Moreover, homogeneous price categories may help firms solve coordination problems and may make the market more transparent for producers, facilitating reciprocal monitoring and thus making collusion easier to sustain.

However, one must take into account that homogeneous price categories may also facilitate price comparisons by consumers, thereby increasing transparency on the consumer side and making the market more competitive. Besides reducing one-shot equilibrium prices and profits, improved consumer side transparency may make collusion harder to maintain. Hence, if the latter effects dominates, firms may prefer to commit to different price categories in order to put some friction back into the market and "obfuscate" consumers. Moreover, this argument points out that whenever homogeneous price categories arise (for instance, in a static model), there might be a welfare-improving effect due to the reduction of search costs.

5 Summary, and implications for competition policy

This Report has dealt with the question of why firms might want to choose the same price categories. We have identified different explanations for this phenomenon, some being the product of innocent behavior by the firms, some possibly being the product of anti-competitive behavior.

More particularly, we have seen in Section 3 that homogeneity of price categories might simply be the natural outcome of normal competitive interactions among firms. For instance, firms experiencing similar demand and technological conditions will tend to use the same price categories. Further, history, habits, and the attempt of giving more effective incentive schemes to workers might also explain the rising of homogenous price categories in a given sector.

Nevertheless, our analysis has also showed that in some circumstances firms might have an incentive to adopt the same price categories so as to facilitate collusion. For instance, firms that are asymmetric (in the sense that they enjoy strongest market positions in different markets) would choose different price categories in an environment where collusion is impossible (such as a one-shot market game), but might want to choose the same price categories in an environment where collusion might arise (such as an infinitely repeated market game). This is because, as we have showed in Section 4, homogenous price categories might facilitate collusion (both in the sense of making enforcement of a given collusive outcome more likely, and of rendering coordination on a given collusive outcome simpler).

The possibility that, in the real world, homogeneity of price categories be the result of the coordination of firms that want to better sustain a collusive outcome, cannot therefore be excluded. Nevertheless, the mere existence of such a possibility does not authorize taking an unqualified tough stance against firms that choose the same price categories. For instance, *per se* rules against homogeneity of price categories would be entirely out of question: as we have seen, there are a number of genuine competitive reasons that would push firms to adopt the same price categories.

Furthermore, it is not even clear that Competition Authorities should be more worried about industries where firms choose the same price categories than about those where they are different. Indeed, the fact that firms have the same price categories renders the market more transparent on the demand side, thus favoring shopping around of consumers, and probably rendering the market more competitive.

Although theory has not said the last word on the issue of whether price transparency tends to make collusion more likely (by allowing rival firms to better monitor each other, and

thus policing a collusive outcome), or competition more aggressive (by allowing consumers to better compare price offers, and thus giving firms higher incentives for aggressive price behavior), empirical evidence (that we review in Section 4.4.2) does show that in markets where consumers are more informed about prices, price levels are lower.

If anything, therefore, we should expect that if we could divide all markets into two subsets, those characterized by similar enough price categories, and all others (where price categories are sufficiently different), price levels should be on average lower in the former subset.

This suggests that - ceteris paribus - homogenous price categories should not make the object of particular attention from the Competition Authorities.

Still, we acknowledge that it is possible that firms might want to adopt the same price categories as a way to enforce collusion. Since at the moment we cannot provide any guidelines as to under which observable circumstances firms are more likely to choose homogeneity in price categories for anti-competitive reasons, the only sensible rule to adopt is that an explicit agreement among firms to use the same price categories should be unlawful, whereas if firms arrived independently at the same outcome, it should be lawful.

In other words, an *agreement* among firms to choose the same price categories should be considered as a facilitating practice and fall under the scope of article 81 of the Treaty (or article 2 of the Italian Law), even absent hard evidence on a price agreement, a similar treatment as when firms exchange disaggregated information on prices and/or outputs.

6 Appendix: Formalization of (some of) the arguments made in the report

In this technical Appendix we formalize some of the arguments that are non-technically presented in the text of the Report. Sections 6.1 and 6.2 address the main questions asked in this Report, namely what drives firms to select the same price categories: they deal respectively with non-collusive and with collusive motives, and their results have been summarized and discussed respectively in Sections 3 and 4 of the Report.

Sections 6.3 and 6.4 endogenize the choice of price categories (which is exogenously given in the previous technical sections) and therefore formalize the arguments reported in Sections 2.2.4 and 2.2.3 respectively.[23]

6.1 Static model (one-shot market interaction)

6.1.1 The Model

We consider two firms A and B which sell in three markets $i = 1, 2, 3$. We analyze two firms and three markets to keep the model as simple as possible, but the same qualitative results would be obtained if we considered n generic markets and s firms. In other words, dealing with two firms and three markets is the easiest possible setting within which we can analyze the problem at hand. (Recall that we are interested in studying the effects of different firms adopting the same product categories, that is the case where they choose to set a unique price for more than one product or market. Therefore, having only two markets would not help: if firms choose to set a unique price for more than one market, they would necessarily have the same price on the same and only two markets.)

Markets should be interpreted in a wide sense, and they might be thought of both as geographical markets or as group of consumers. For instance, we could think of firms A and B as airlines serving one given route at different times or with different in-flight services (this would be the source of differentiation among their products that we discuss below), and facing three different groups of consumers (say, business travellers, adults, young travellers); or of two different music firms having different authors/orchestras in their catalogues and selling records of three different genres (say, jazz, classic, rock); or two cinemas located in

[23] The order of the arguments made in this Appendix does not follow by choice the order of the Sections of the text. We deal first with the two models that focus on the main topic of the Report (why firms choose the *same* price categories), and Sections 6.3 and 6.4 (which deal with why firms might use price categories at all) are inverted relative to the order followed in the text to facilitate the reading (the search cost model is completely different from the rest of the models used here).

two different areas of town and showing the same three films in their three film-rooms, and so on.

The two firms might face different willingness to pay in the different markets, which is meant to reflect possible asymmetries among the firms in the different markets considered. This translates into saying that v_{iA} might differ from v_{iB}, and that a priori v_{iA} can differ from v_{jA} for any $i, j = 1, 2, 3$ with $i \neq j$. (For instance, business travellers might have a higher willingness to pay for airline A than for airline B because the former airline's flights leave early in the morning and the latter's in the middle of the day, whereas young people might prefer airline B precisely for the same reason. Or cinema A might systematically attract a larger audience to their children movies because it is located closer to a residential part of town, whereas cinema B attracts a larger audience of *films d'essai* because located closer to university.)

In each market i, firms A and B sell the same products which are perceived with the same degree of substitution m by consumers in any market $i = 1, 2, 3$. This is just a simplifying assumption (allowing for different parameters of substitution would considerably complicate the analysis without adding any insight).

The demand function faced by firms A and B in each group of consumers/market i are:

$$q_{iA} = \frac{1}{2}\left(v_{iA} - p_{iA}\left(1 + \frac{m}{2}\right) + \frac{m}{2}p_{iB}\right) \tag{1}$$
$$q_{iB} = \frac{1}{2}\left(v_{iB} - p_{iB}\left(1 + \frac{m}{2}\right) + \frac{m}{2}p_{iA}\right) \tag{2}$$

where $m \in [0, \infty)$ is the parameter of product substitutability between firm A and B.[24]

We assume that the two firms have identical constant marginal costs, that we normalize to zero without any loss of generality, and no fixed costs of production. Again, this is for simplicity and it will allow us to focus on the main elements driving the results. (Note that we could have also assume that firms face the same identical willingness to pay in the different markets, but differ in production costs across markets. We would obtain exactly the same insights. What would complicate the analysis without adding insight is considering different sources of potential asymmetries, such as both demand and cost differences, at the same time.)

[24] These demand functions can be derived by the following utility function of each group /market i of consumers:

$U_i = \frac{v_{iA}}{1+m}q_{iA} + \frac{v_{iB}}{1+m}q_{iB} + \frac{m(v_{iA}+v_{iB})}{2(1+m)}(q_{iA} + q_{iB}) - \frac{1}{(1+m)}\left[q_{iA}^2 + q_{iB}^2 + \frac{m}{2}(q_{iA} + q_{iB})^2\right] + y,$

with y being an outside good. Their advantage is that as m tends to infinity, there are no discontinuities in the asymmetric case (despite a difference in valuations, both firms will sell). These demand functions are a variant of the functions first proposed by Shubik and Levitan (1980).

Endowed with this simple model, we should now look at whether firms want to set the same price for more than one product, and if so which products they would put in the same price category. For the purpose of clarity, we introduce the following definitions that we shall use throughout the report.

Definition 1 *We call* ***price category*** *jk a situation where a given firm S $(= A, B)$ commits to sell at the same price p_{jkS} $(j, k = 1, 2, 3;\ j \neq k)$ in the two markets j and k (with the third market being priced independently).*

Definition 2 *We call (full)* ***price discrimination*** *when a given firm S $(= A, B)$ does not restrict itself in its price choice, and is therefore free to sell in all the markets at different prices p_{jS} $(j = 1, 2, 3)$.*

Definition 3 *We call* ***price uniformity*** *when a given firm commits to sell at the same price p in all the three markets.*

6.1.2 Price categories v. price discrimination: best reply analysis

In what follows, we take as given the price choice of one firm, and we look at what are the optimal price choices of the other firm. Obviously, this does not give the proper equilibrium of the game played by the two firms, but will give us important insights on the issues we are set to study.

(Full) price discrimination Suppose we are interested in looking at the optimal behavior of firm A for given price choices of firm B in all three markets. In other words, in this paragraph we take as given p_{1B}, p_{2B}, and p_{3B}, and we look for optimal choices of firm A only. When firm A does not restrict its price choices, i.e. if it fully price discriminates, its profits will be given by:

$$\pi_A = \sum_{i \in 1,2,3} \frac{1}{2} \left(v_{iA} - p_{iA} \left(1 + \frac{m}{2} \right) + \frac{m}{2} p_{iB} \right) p_{iA}, \tag{3}$$

where in any market i, p_{iB} may or may not coincide with p_{jB} (for $i, j = 1, 2, 3$, and $i \neq j$). By setting $\partial \pi_A / \partial p_{iA} = 0$, we obtain the FOCs of firm A:

$$FOC_{iA} : v_{iA} - p_{iA} (2 + m) + \frac{m}{2} p_{iB} = 0, \tag{4}$$

which can be rewritten as the best response functions, one for each market:

$$BR_{iA} (p_{iB}) : p_{iA} = \frac{2 v_{iA} + m p_{iB}}{2(2 + m)}. \tag{5}$$

Given the prices set by firm B, therefore, by optimally choosing its prices firm A will be able to obtain the following profits if it engages in full price discrimination:

$$\pi_A^{pd}(p_{1B}, p_{2B}, p_{3B}) = \sum_{i=1,2,3} \frac{(2v_{iA} + mp_{iB})^2}{16(2+m)}. \tag{6}$$

Price categories Suppose now that we continue to take as given the price choices of firm B, and we wonder now what would be the optimal price choices of firm A if it were to adopt price categories, that is if it committed to sell more than one product at the same price. Suppose for instance that firm A decided to commit and have the same price p_{jkA} in markets j and k. Its profits will be:

$$\pi_A = \sum_{i=j,k} \frac{1}{2}\left(v_{iA} - p_{jkA}\left(1+\frac{m}{2}\right) + \frac{m}{2}p_{iB}\right)p_{jkA} + \frac{1}{2}\left(v_{lA} - p_{lA}\left(1+\frac{m}{2}\right) + \frac{m}{2}p_{lB}\right)p_{lA}. \tag{7}$$

The *two* FOCs will be given by $\partial\pi_A/\partial p_{jkA} = 0$ and $\partial\pi_A/\partial p_{lA} = 0$, which can be written:

$$FOC_{jkA} : 2(v_{jA} + v_{kA}) - 4p_{jkA}\,(2+m) + m(p_{jB} + p_{kB}) = 0, \tag{8}$$

$$FOC_{lA} : v_{lA} - p_{lA}\,(2+m) + \frac{m}{2}p_{lB} = 0, \tag{9}$$

From which we obtain the two best response functions:

$$BR_{jkA}\,(p_{jB}, p_{kB}) : p_{jkA} = \frac{2(v_{jA} + v_{kA}) + m(p_{jB} + p_{kB})}{4(2+m)}, \tag{10}$$

$$BR_{lA}\,(p_{lB}) : p_{lA} = \frac{2v_{lA} + mp_{lB}}{2(2+m)}. \tag{11}$$

For given prices of firm B, and by substitution, we can then find the highest profits that firm A can attain if it commits to set the same price in markets j and k:

$$\pi_A^{jk}(p_{jB}, p_{kB}, p_{lB}) = \frac{(2(v_{jA} + v_{kA}) + m(p_{jB} + p_{kB}))^2}{32(2+m)} + \frac{(2v_{lA} + mp_{lB})^2}{16(2+m)}. \tag{12}$$

Profit comparison: better price discrimination It is now easy to see that by choosing to adopt the same price for two separate markets, firm A will lose profits. Indeed,

$$\pi_A^{pd}(\cdot) - \pi_A^{jk}(\cdot) = \frac{(2(v_{jA} - v_{kA}) + m(p_{jB} - p_{kB}))^2}{32(2+m)} \geq 0, \tag{13}$$

the two profit expressions coinciding if $v_{jA} = v_{kA}$ and $p_{jB} = p_{kB}$, or if the numerator equals zero.

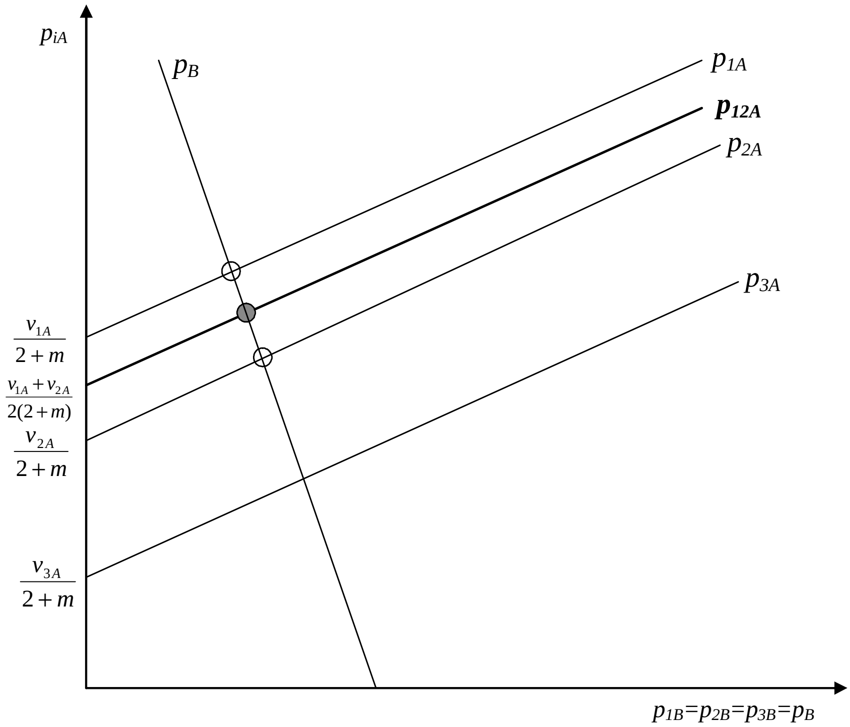

Figure 1: *Best replies. The thick line represents the best reply of firm A when it commits to have the same price in market 1 and 2.*

To sum up, this little exercise tells us that - given the price choices of the rival - a firm always prefers to price discriminate. The intuition is simple, and coincides with the usual explanation of why firms find it more profitable to price discriminate rather than to set uniform prices: by price discriminating the firm has a higher degree of freedom, and can better target the price to different markets.

Figure 1 might be instructive in this respect. The Figure is drawn for the case where $v_{1A} > v_{2A} > v_{3A}$ and $p_{1B} = p_{2B} = p_{3B} = p_B$. In the plane (p_B, p_{iA}) it illustrates the best responses of firm A when it fully price discriminates (thin lines) and when it commits to fix the same price in markets 1 and 2 (thick line). In the latter case, firm A averages between the full discrimination best replies in the two markets. When firm A is free to sell in all the markets at different prices, in equilibrium it will set a high price in its high valuation market (market 1) and a lower one in its intermediate valutation market (market 2). Instead, when it restricts its price choice bundling market 1 and 2, it has to choose a sub-optimal price which lies between the two.

If price categories, which one is optimal? The model we have proposed is therefore one which suggests - at least so far in the static version - that if a firm could, it would choose to fully price discriminate. Nevertheless, full price discrimination might not be profitable for a series of reasons, such as transaction costs, menu costs, or consumers' search costs, as discussed in Section 2.

For instance, suppose that there exist menu costs that are so important that a firm would not want to set three different prices, but only two of them. Which markets would the firm choose to sell in at the same price? In other words, if a price category jk has to be, what determines the choice of a firm about which markets j and k should be served at the same price p_{jk}?

The previous example, where prices set by the rival firm are given, might help us gain some insight in this question. Assume for instance, without loss of generality, that:

$$\textit{Assumption A1:} \qquad v_{1A} \geq v_{2A} \geq v_{3A}. \tag{14}$$

To simplify matters, we also make the following:

$$\textit{Assumption A2:} \quad \textit{each firm serves all markets} \tag{15}$$

Which price category will firm A choose? Let us see under which condition, for instance, firm A will prefer to set a common price in markets 1 and 2 rather than in any other pair of markets. For this to be optimal, it must be:

$$\begin{gathered}\pi_A^{12}(\cdot) - \pi_A^{13}(\cdot) = \\ \frac{\left[2(v_{2A} - v_{3A}) + m(p_{2B} - p_{3B})\right]\left[2(2v_{1A} - v_{2A} - v_{3A}) + m(2p_{1B} - p_{2B} - p_{3B})\right]}{32(2+m)} \geq 0\end{gathered} \tag{16}$$

and

$$\begin{gathered}\pi_A^{12}(\cdot) - \pi_A^{23}(\cdot) = \\ \frac{\left[2(v_{1A} - v_{3A}) + m(p_{1B} - p_{3B})\right]\left[2(2v_{2A} - v_{1A} - v_{3A}) + m(2p_{2B} - p_{1B} - p_{3B})\right]}{32(2+m)} \geq 0.\end{gathered} \tag{17}$$

It is difficult to draw general conclusions from these two inequalities without imposing assumptions on the prices charged by firm B. However, to gain some insights into this issue, suppose that $p_{1B} = p_{2B} = p_{3B}$. In this case, it is easy to see that the first inequality is always satisfied, and that the second is satisfied as long as $v_{2A} \geq (v_{1A} + v_{3A})/2$. This tells us that,

at least when the rival firm regards the three markets as symmetric, firm A will choose to set a common price in the markets which are more similar to each other (the above mentioned condition says that market 2 demand is closer to market 1 demand than it is to market 3 demand).

Intuitively, this result has a simple explanation. Firm A would like to set different prices in each market, so as to reflect their different marginal profitabilities; if it has to set a common price in a pair of markets, this entails a distortion from the first best, and such a distortion will be the lower the closer the markets will be to each other. In the limit, and again for given equal prices of the rival firm, it is clear that firm A would not lose anything by setting a common price for any markets j and k if they were identical, whereas the loss would be maximal if j and k were the most different to each other.

The considerations above carry over also to the case where the rival firm has the same ranking of markets (recall that, other things being equal, the higher the willingness to pay in a market the higher the equilibrium price). However, this might not be the case if firm B had a very different ranking (which would translate into higher prices in the markets where firm A has lowest demand).

Note also that the assumption that firm A serves all three markets might not be completely innocuous here. For instance, suppose that $v_{3A} \to 0$. In this case, firm A might find it optimal to set the common price for markets 1 and 3: it would simply choose p_{13A} as the price which maximizes its profit in market 1 and would not sell anything in market 3, but this would not be costly because sales in that market would be irrelevant anyhow.

But to be able to analyze properly these issues, it is necessary to move to the full equilibrium analysis, that considers firm B's actions as well.

6.1.3 Price categories: the full game

Let us now study the following game. In the first stage, firms independently and simultaneously commit on a certain pricing policy. Each firm can fully price discriminate, or set a common price for any pair of markets, or choose uniform pricing, i.e., set a common price in all three markets.

In the second stage of the game, firms choose prices according to the policy they have committed to in the previous period, and profits are realized.

We look for the sub-game perfect equilibrium of this game, and as usual we solve it by backward induction. Accordingly, the first thing to do is to look for the equilibrium profits for all the different price configurations that may arise from firms' choices at stage 1. To

be precise, there exist (5x5) cases, for a total of 25 combinations of profits that one should compute, although some differ only in their labels. In the following sub-section, we derive the equilibrium prices and profits that correspond to the different combinations.

Equilibria of the sub-games In what follows, we report the price equilibria for the most important price configurations. First, mostly as they give us a benchmark, we report the equilibria for the cases where the firms can (fully) price discriminate and where they both choose price uniformity. Then, we look at all the equilibria which involve price categories by at least one firm. (For shortness, and because from previous analysis we already know they will not be chosen at equilibrium, we omit derivations of equilibria involving price uniformity.)[25]

(Full) price discrimination equilibrium When firms do not restrict their price choices, i.e. they fully price discriminate, their profits will be given by:

$$\pi_A = \sum_{i \in 1,2,3} \frac{1}{2} \left(v_{iA} - p_{iA} \left(1 + \frac{m}{2} \right) + \frac{m}{2} p_{iB} \right) p_{iA}, \tag{18}$$

and

$$\pi_B = \sum_{i \in 1,2,3} \frac{1}{2} \left(v_{iB} - p_{iB} \left(1 + \frac{m}{2} \right) + \frac{m}{2} p_{iA} \right) p_{iB}. \tag{19}$$

First order conditions are given by $\partial \pi_S / \partial p_{iS} = 0$, or:

$$FOC_{iA} : v_{iA} - p_{iA} \left(2 + m \right) + \frac{m}{2} p_B = 0, \tag{20}$$

and

$$FOC_{iB} : v_{iB} - p_{iB} \left(2 + m \right) + \frac{m}{2} p_A = 0, \tag{21}$$

Each FOC defines a best response function. For instance, the best response function of firm A in market i is given by:

$$BR_{iA} \left(p_{iB} \right) : p_{iA} = \frac{2v_{iA} + mp_{iB}}{2(2+m)}. \tag{22}$$

By solving the system of the FOCs we obtain the equilibrium prices in each market i as:

$$p_{iA}^{*} = \frac{2 \left(2v_{iA}(m+2) + mv_{iB} \right)}{(m+4)(3m+4)} \tag{23}$$

$$p_{iB}^{*} = \frac{2 \left(2v_{iB}(m+2) + mv_{iA} \right)}{(m+4)(3m+4)}. \tag{24}$$

[25] They are also the easiest to compute, so the interested reader can find those equilibria by him/herself.

Note that

$$p^*_{iA} - p^*_{iB} = \frac{2}{3m+4}\left(v_{iA} - v_{iB}\right), \tag{25}$$

from which it is immediate to see that - not surprisingly - the firm which faces a larger demand intercept (that is, the firm whose consumers show a higher willingness to pay) will set a higher price at equilibrium. By substitution it is easy to obtain the profits that each firm obtains in a market i:

$$\pi^*_{iA} = \frac{(2+m)}{(m+4)^2(3m+4)^2}\left(2v_{iA}(m+2) + v_{iB}m\right)^2 \tag{26}$$

$$\pi^*_{iB} = \frac{(2+m)}{(m+4)^2(3m+4)^2}\left(2v_{iB}(m+2) + v_{iA}m\right)^2 \tag{27}$$

Note also that if the firms' demands are symmetric in a market i, that is if $v_{iA} = v_{iB}$, then - again unsurprisingly - the firms will set the same price at equilibrium, and will receive identical profits:

$$p^*_{iA} = p^*_{iB} = \frac{2v_i}{m+4}; \quad \pi^*_{iA} = \pi^*_{iB} = v_i^2 \frac{2+m}{(m+4)^2} \tag{28}$$

Uniform pricing equilibrium At the other extreme, when firms maximally restrict their price choices, i.e. they choose to set a uniform price in all markets, their profits will be given by:

$$\pi_A = \sum_{i\in 1,2,3} \frac{1}{2}\left(v_{iA} - p_A\left(1 + \frac{m}{2}\right) + \frac{m}{2}p_B\right)p_A, \tag{29}$$

and

$$\pi_B = \sum_{i\in 1,2,3} \frac{1}{2}\left(v_{iB} - p_B\left(1 + \frac{m}{2}\right) + \frac{m}{2}p_A\right)p_B. \tag{30}$$

First order conditions are given by $\partial\pi_S/\partial p_S = 0$ (with $S = A, B$), and from them the best responses can be found as:

$$BR_A(p_B) \ : \ p_A = \frac{2\sum_{i=1,2,3} v_{iA} + mp_B}{6(2+m)} \tag{31}$$

$$BR_B(p_A) \ : \ p_B = \frac{2\sum_{i=1,2,3} v_{iB} + mp_A}{6(2+m)} \tag{32}$$

By solving the system of the best response functions we obtain the equilibrium uniform prices as:

$$p^*_{123A} = \frac{2\left(2(m+2)\sum_{i=1,2,3} v_{iA} + m\sum_{i=1,2,3} v_{iB}\right)}{3(m+4)(3m+4)} \tag{33}$$

$$p^*_{123B} = \frac{2\left(2(m+2)\sum_{i=1,2,3} v_{iB} + m\sum_{i=1,2,3} v_{iA}\right)}{3(m+4)(3m+4)}. \tag{34}$$

By substitution it is easy to obtain the profits of each firm as:

$$\pi^*_{123A} = \frac{(2+m)}{3\left(m+4\right)^2\left(3m+4\right)^2}\left(2\left(m+2\right)\sum_{i=1,2,3} v_{iA} + m\sum_{i=1,2,3} v_{iB}\right)^2 \tag{35}$$

$$\pi^*_{123B} = \frac{(2+m)}{3\left(m+4\right)^2\left(3m+4\right)^2}\left(2\left(m+2\right)\sum_{i=1,2,3} v_{iB} + m\sum_{i=1,2,3} v_{iA}\right)^2. \tag{36}$$

Price categories (when all markets are served): equilibrium outcomes When firms set price categories, we might have two cases: (a) symmetric configurations, namely when firms choose to set a common price in the same markets j and k; (b) asymmetric configurations, when a firm sets a common price in markets j and k, while the other sets a common price in markets j and l.

(a) symmetric configurations In this configuration, we have two different cases according to whether we are looking at *(a1)* the market whose price is independent of other markets or *(a2)* at the markets in the same price category.

(a1) If, say, firms A and B have the same price category jk, then in market l we shall have:

$$BR_{lA}\left(p_{lB}\right) \ : \ p_{lA} = \frac{2v_{lA} + mp_{lB}}{2(2+m)}, \tag{37}$$

$$BR_{lB}\left(p_{lA}\right) \ : \ p_{lB} = \frac{2v_{lB} + mp_{lA}}{2(2+m)}. \tag{38}$$

Equilibrium prices will be:

$$p^*_{lA} = \frac{2\left(2v_{lA}(m+2) + mv_{lB}\right)}{(m+4)(3m+4)} \tag{39}$$

$$p^*_{lB} = \frac{2\left(2v_{lB}(m+2) + mv_{lA}\right)}{(m+4)(3m+4)}, \tag{40}$$

and by substitution profits are:

$$\pi^*_{lA} = \frac{(2+m)}{(m+4)^2(3m+4)^2}\left(2v_{lA}(m+2)+v_{lB}m\right)^2 \quad (41)$$

$$\pi^*_{lB} = \frac{(2+m)}{(m+4)^2(3m+4)^2}\left(2v_{lB}(m+2)+v_{lA}m\right)^2 \quad (42)$$

(a2) In the two markets j and k with a common price, firms will have the following best response functions:

$$BR_A(p_{jkB}) : p_{jkA} = \frac{(v_{jA}+v_{kA})+mp_{jkB}}{2(2+m)} \quad (43)$$

$$BR_B(p_{jkA}) : p_{jkB} = \frac{(v_{jB}+v_{kB})+mp_{jkA}}{2(2+m)} \quad (44)$$

Equilibrium:

$$p^*_{jkA} = \frac{2(2+m)(v_{jA}+v_{kA})+m(v_{jB}+v_{kB})}{(m+4)(3m+4)} \quad (45)$$

$$p^*_{jkB} = \frac{2(2+m)(v_{jB}+v_{kB})+m(v_{jA}+v_{kA})}{(m+4)(3m+4)} \quad (46)$$

$$\pi^*_{jA}+\pi^*_{kA} = \frac{2+m}{2(m+4)^2(3m+4)^2}\left(2(m+2)(v_{jA}+v_{kA})+m(v_{jB}+v_{kB})\right)^2 \quad (47)$$

$$\pi^*_{jB}+\pi^*_{kB} = \frac{2+m}{2(m+4)^2(3m+4)^2}\left(2(m+2)(v_{jB}+v_{kB})+m(v_{jA}+v_{kA})\right)^2 \quad (48)$$

Note that

$$(\pi^*_{jA}+\pi^*_{kA})-(\pi^*_{jB}+\pi^*_{kB}) = \frac{2+m}{2(m+4)(3m+4)}\left((v_{jA}+v_{kA})^2-(v_{jB}+v_{kB})^2\right) \quad (49)$$

Total profits across all markets are thus:

$$\pi^*_A(jk,jk) = \frac{(2+m)}{(m+4)^2(3m+4)^2}\left((2(m+2)v_{lA}+mv_{lB})^2+\frac{1}{2}\left(2(m+2)(v_{jA}+v_{kA})+m(v_{jB}+v_{kB})\right)^2\right) \quad (50)$$

$$\pi^*_B(jk,jk) = \frac{(2+m)}{(m+4)^2(3m+4)^2}\left((2(m+2)v_{lB}+mv_{lA})^2+\frac{1}{2}\left(2(m+2)(v_{jB}+v_{kB})+m(v_{jA}+v_{kA})\right)^2\right). \quad (51)$$

(b) Asymmetric configurations Suppose first that firm A sets a common price for markets j and k, whereas firm B fully price discriminates. In this case, the best response functions are:

$$BR_{jkA}(p_{jB},p_{kB}) : p_{jkA} = \frac{2(v_{jA}+v_{kA})+m(p_{jB}+p_{kB})}{4(2+m)}, \quad (52)$$

and

$$BR_{iB}\left(p_{jkA}\right): p_{lB} = \frac{2v_{iB} + mp_{jkA}}{2(2+m)}, \text{ with } i = j, k, \tag{53}$$

the best responses in the remaining (independent) markets being as above.

Suppose instead that firm A sets a common price for markets j and k, whereas firm B sets a common price in markets k and l. In this case, the best response functions are:

$$BR_{jkA}: p_{jkA} = \frac{(2(v_{jA} + v_{kA}) + m(p_{klB} + p_{jB})}{2(2+m)} \tag{54}$$

$$BR_{lA}: p_{lA} = \frac{(2(v_{lA}) + m(p_{klB})}{2(2+m)} \tag{55}$$

$$BR_{klb}: p_{klB} = \frac{(2(v_{kB} + v_{lB}) + m(p_{jkA} + p_{lA})}{2(2+m)} \tag{56}$$

$$BR_{jB}: p_{jB} = \frac{(2(v_{jB}) + m(p_{jkA})}{2(2+m)}. \tag{57}$$

Solving these best reply functions one obtains the equilibrium prices and profits. However, the resulting expressions are extremely lengthy and little can be gained from inspecting them. Since, additionally, we are interested in comparing profits made by the firms under the different configurations, and such comparisons are difficult to analyze in general, we restrict the parameters space and carry out a full equilibrium analysis for two relevant cases: (i) firms are symmetric, in the sense they are ranked by consumers in the same way in the different markets; (ii) firms are very asymmetric, in the sense that the market where one firm is strongest is the market where the rival is weakest and vice versa.

6.1.4 Example (i): symmetric firms (all markets are served)

We assume in this first example that firms are ranked in the same way by consumers belonging to the same market: $v_{1A} = v_{1B} = v_1$, $v_{2A} = v_{2B} = v_2$, and $v_{3A} = v_{3B} = v_3$, with $v_1 > v_2 > v_3$, and $v_2 > (v_1 + v_3)/2$ (that is, market 2 is more similar to market 1 than market 3).

The analysis above can be used to derive the relevant equilibrium profits for each of the pairs of price configurations we should look at.

Notation 4 *We denote price configurations by indicating the markets for which a firm sets a common price. For instance, $\pi_A(jk, \cdot)$ refers to the profits made by firm A when firm A puts markets j and k in the same price category, and firm B price discriminates, or $\pi_B(jkl, jk)$*

refers to the profits made by firm B when firm A puts markets j, k and l in the same price category (that is, it adopts uniform pricing), and firm B puts markets j and k in the same price category.

In what follows, we shall show that when firms are symmetric (in the sense defined above) and they cannot fully discriminate, the only equilibrium of the two-stage game where they first choose price categories and then choose prices (for given price categories) is given by $(12, 12)$, that is, they end up choosing the same price categories (more precisely, they set a common price in their most important markets). To show this, we shall proceed as follows. First, we show that given that firm B chooses price category (12), the best reply of firm A is also (12). This, given symmetry, is enough to prove that $(12, 12)$ is an equilibrium of the game. Next, we show that there is no other equilibrium of the game.

Let us now look at the profits *when firm B sets a common price in markets* 1 *and* 2, and look at all the different combinations according to the price choices made by firm A in the previous period.

If firm A fully discriminates, equilibrium profits are:

$$\pi_A^*(\cdot, 12) = \frac{(32 + 24m + 5m^2)(v_1^2 + v_2^2) + 2m(8 + 3m)(v_1 + v_2) + 8(2 + m)^2 v_3^2}{8(2 + m)\,(m + 4)^2} \tag{58}$$

$$\pi_B^*(\cdot, 12) = \frac{(2 + m)((v_1 + v_2)^2 + 2v_3^2)}{2\,(m + 4)^2}. \tag{59}$$

When both firms set a common price in markets 1 and 2, equilibrium profits are:

$$\pi_A^*(12, 12) = \frac{(2 + m)((v_1 + v_2)^2 + 2v_3^2)}{2\,(m + 4)^2}. \tag{60}$$

$$\pi_B^*(12, 12) = \frac{(2 + m)((v_1 + v_2)^2 + 2v_3^2)}{2\,(m + 4)^2}. \tag{61}$$

Next, we should look at the asymmetric configurations. For instance, the case where firm A sets a common price in markets 1 and 3, whereas firm B puts markets 1 and 2 in the same price category. (Similarly, one can obtain the equilibrium outcomes for the case where firm A sets a common price in markets 2 and 3.)

By solving the best reply functions found in the previous section, we obtain the following equilibrium prices

$$p_{2A}(13, 12) = \frac{2\left[64v_2 + 8m(v_1 + 7v_2) + m^2(3v_1 + 11v_2 + v_3)\right]}{256 + 320m + 124m^2 + 15m^3} \tag{62}$$

$$p_{13A}(13,12) = \frac{2\left[(32+28m+6m^2)v_1 + (32+32m+7m^2)v_3 + 2m(2+m)v_2\right]}{256+320m+124m^2+15m^3} \tag{63}$$

$$p_{3B}(13,12) = \frac{2\left[64v_3 + 8m(v_1+7v_3) + m^2(3v_1+11v_3+v_2)\right]}{256+320m+124m^2+15m^3} \tag{64}$$

$$p_{12B}(13,12) = \frac{2\left[(32+28m+6m^2)v_1 + (32+32m+7m^2)v_2 + 2m(2+m)v_3\right]}{256+320m+124m^2+15m^3}. \tag{65}$$

Equilibrium profits under this pair of pricing configurations will be:

$$\pi_A(13,12) =$$
$$\frac{(2+m)\left[(8+3m)^2(32+32m+9m^2)v_1^2+(4096+7168m+4576m^2+1264m^3+129m^4)v_2^2+2m(256+448m+240m^2+39m^3)v_2v_3\right]}{(8+3m)^2(32+28m+5m^2)^2}+$$
$$+\frac{(2+m)\left[(2048+4096m+2944m^2+896m^3+99m^4)v_3^2+2(8+3m)v_1(256v_3+96m(v_2+4v_3)+8m^2(11v_2+23v_3)+m^3(19v_2+29v_3))\right]}{(8+3m)^2(32+28m+5m^2)^2}. \tag{66}$$

And, *mutatis mutandis*, one could also write down the equilibrium profits of firm B, as well as the equilibrium prices and profits when firm A sets a common price on markets 2 and 3.

Finally, we should find equilibrium prices and profits when firm A does uniform pricing whereas firm B sets a common price for markets 1 and 2. By taking the FOCs and solving them, we obtain:

$$p_{123A}(123,12) = \frac{2\left[v_1+v_2+v_3)\right]}{3(4+m)} \tag{67}$$

$$p_{12B}(123,12) = \frac{(12+5m)(v_1+v_2)+2mv_3)}{6(2+m)(4+m)} \tag{68}$$

$$p_{3B}(123,12) = \frac{(12+4m)v_3+m(v_1+v_2)}{3(2+m)(4+m)} \tag{69}$$

As for the profits, they are:

$$\pi_A^*(123,12) = \frac{(2+m)(v_1+v_2+v_3)^2}{3\,(m+4)^2}. \tag{70}$$

Proposition 1 *If full price discrimination is not possible, then an equilibrium at the first stage of the game involves both (symmetric) firms choose the same price category for markets* 1 *and* 2 *(which are the most similar to each other).*

Proof. To prove this proposition, it is enough to check the best reply of firm A when firm B sets a common price on markets 1 and 2. First, it is easy to see that firm A prefers to set a price category (12) rather than to do uniform pricing:

$$\pi_A^*(12, 12) - \pi_A^*(123, 12) = \frac{(2+m)(v_1 + v_2 - 2v_3)^2}{6\,(m+4)^2} > 0. \tag{71}$$

Next, one can show that A is better off setting a common price in markets (12) rather than in markets (13):

$$\pi_A^*(12, 12) - \pi_A^*(13, 12) > 0. \tag{72}$$

This inequality turns out to be quite a lengthy expression. Numerical analysis shows it always holds. For the sake of illustration, we reproduce here its values under a pair of specific examples. Let us normalize $v_1 = 1$, and fix $v_2 = 3/4$ and $v_3 = 1/4$. Then the corresponding inequality becomes:

$$\frac{(2+m)(2048 + 4032m + 2924m^2 + 934m^3 + (1767m^4)/16)}{2(4+m)^2(8+3m)^2(8+5m)^2} > 0 \tag{73}$$

Results do not change under different values of v_2 and v_3 which satisfy the hypothesis $v_2 > (v_1 + v_3)/2$. For instance, if $v_1 = 1$, $v_2 = 3/4$ and $v_3 = 1/8$, the inequality becomes:

$$\frac{(2+m)(3840 + 7552m + 5488m^2 + 1760m^3 + 209m^4)}{8(4+m)^2(8+3m)^2(8+5m)^2} > 0. \tag{74}$$

Finally, one can also check that firm A prefers (12) to (23):

$$\pi_A^*(12, 12) - \pi_A^*(23, 12) > 0. \tag{75}$$

Again, this inequality turns out to be a very lengthy expression (that, however, is easily taken care of by computer packages such as Mathematica and Maple). For the sake of illustration, let us reproduce the results for two examples. If $v_1 = 1$, $v_2 = 3/4$ and $v_3 = 1/4$, the inequality becomes:

$$\frac{(24576 + 69632m + 78720m^2 + 44032m^3 + 12078m^4)}{32(4+m)^2(8+3m)^2(8+5m)^2} > 0. \tag{76}$$

If $v_1 = 1$, $v_2 = 3/4$ and $v_3 = 1/8$, the inequality becomes:

$$\frac{3(3584 + 9728m + 10592m^2 + 5744m^3 + 1538m^4 + 161m^5)}{8(4+m)^2(8+3m)^2(8+5m)^2} > 0. \tag{77}$$

However, it is also important to note that a crucial assumption for the proposition to hold is that full price discrimination is not allowed. Indeed, if firm A could price discriminate, then it would choose to do so:

$$\pi_A^*(\cdot, 12) - \pi_A^*(123, 12) = \frac{(v_1 - v_2)^2}{8\,(m+2)^2} > 0. \tag{78}$$

■

To conclude, we have showed that - if full price discrimination is not allowed - then firms, that here are ranked in the same way by consumers in each market, would choose to put the same two products in the same price category: (12) would be the best response to (12).

If full price discrimination was allowed, then one could show both that (i) full discrimination would be the dominant strategy for each firm, and that (ii) firms would be better off at the equilibrium where both price discriminate.

Proposition 2 *If full price discrimination is not possible, then the equilibrium* $(12, 12)$ *is unique.*

Proof. A possible equilibrium of the game (to be more precise, of the restricted game where full price discrimination is not allowed) is $(13, 13)$. To show this is not an equilibrium, it is sufficient to check that if firm B sets a common price in markets 1 and 3, firm A's best reply is not to set a price category (13), but rather a price category (12). In other words, it is enough to show that $\pi_A^*(12, 13) > \pi_A^*(13, 13)$. In the case where both firms set price category (13), prices and profits are easily found by substituting the expressions already found in the previous subsection:

$$\begin{aligned}
p_{13A}(13, 13) &= p_{13B}(13, 13) = \frac{v_1 + v_3}{4 + m}; && (79)\\
p_{2A}(13, 13) &= p_{2B}(13, 13) = \frac{2v_2}{4 + m}; && (80)\\
\pi_A(13, 13) &= \pi_B(13, 13) = \frac{(2+m)\left[(v_1 + v_3)^2 + 2v_2^2\right]}{2\,(4+m)^2}. && (81)
\end{aligned}$$

In the case where price categories are respectively (12) and (13), the expressions are more complex. The equilibrium prices can be obtained by appropriately replacing the best reply functions found in the previous subsection; they are - mutatis mutandis - identical to the expressions found above for the case $(13, 12)$. They are reproduced here for convenience:

$$p_{2B}(12, 13) = \frac{2\left[64v_2 + 8m(v_1 + 7v_2) + m^2(3v_1 + 11v_2 + v_3)\right]}{256 + 320m + 124m^2 + 15m^3} \tag{82}$$

$$p_{13B}(12,13) = \frac{2\left[(32+28m+6m^2)v_1 + (32+32m+7m^2)v_3 + 2m(2+m)v_2\right]}{256+320m+124m^2+15m^3} \tag{83}$$

$$p_{3A}(12,13) = \frac{2\left[64v_3 + 8m(v_1+7v_3) + m^2(3v_1+11v_3+v_2)\right]}{256+320m+124m^2+15m^3} \tag{84}$$

$$p_{12A}(12,13) = \frac{2\left[(32+28m+6m^2)v_1 + (32+32m+7m^2)v_2 + 2m(2+m)v_3\right]}{256+320m+124m^2+15m^3}. \tag{85}$$

Equilibrium profits under this pair of pricing configurations will be:

$$\begin{aligned}
&\pi_A(12,13) = \\
&\frac{(2+m)\left[(8+3m)^2(32+32m+9m^2)v_1^2+(4096+7168m+4576m^2+1264m^3+129m^4)v_3^2+2m(256+448m+240m^2+39m^3)v_2v_3\right]}{(8+3m)^2(32+28m+5m^2)^2}+ \\
&+\frac{(2+m)\left[(2048+4096m+2944m^2+896m^3+99m^4)v_2^2+2(8+3m)v_1(256v_2+96m(v_3+4v_2)+8m^2(11v_3+23v_2)+m^3(19v_3+29v_2))\right]}{(8+3m)^2(32+28m+5m^2)^2}.
\end{aligned}$$

Numerical solutions show that $\pi_A(12,13) > \pi_A(13,13)$, that is, that firm A would have an incentive to deviate from a candidate equilibrium where both firms choose to set a common price in markets 1 and 3. As an illustration, consider the case where $v_1 = 1$, $v_2 = 1/2$, and $v_3 = 1/4$. The inequality $\pi_A(12,13) - \pi_A(13,13)$ can be rewritten:

$$\frac{(2+m)(32768+50176m+27456m^2+6304m^3+513m^4)}{32(4+m)^2(8+3m)^2(8+5m)^2} > 0. \tag{86}$$

If $v_1 = 1$, $v_2 = 1/2$, and $v_3 = 1/8$. The inequality $\pi_A(12,13) - \pi_A(13,13)$ can be rewritten:

$$\frac{3(2+m)(61440+94208m+51392m^2+11680m^3+931m^4)}{128(4+m)^2(8+3m)^2(8+5m)^2} > 0. \tag{87}$$

Equivalently, one can show that both firms setting a common price in markets 2 and 3 cannot be an equilibrium.

This excludes all possible symmetric equilibria other than (12,12). Symmetric equilibrium are obvious candidates for equilibrium since firms are perfectly symmetric in the model. To be precise, however, we should also prove that there is no asymmetric equilibrium. That there is no equilibrium of the type (12,13) or (12,13) follows from the analysis of the best replies above (when a firm chooses (12) the rival is always better off choosing (12) as well). The remaining possible candidate equilibrium to rule out is (13,23). It is possible to check that this is not an equilibrium either. ■

6.1.5 Example (ii): asymmetric firms

We assume $v_{1A} = v_{3B} = v_1$, $v_{3A} = v_{1B} = v_3$, and $v_{2A} = v_{2B} = v_2$, with $v_1 > v_2 > v_3$, and $v_2 > (v_1 + v_3)/2$ (that is, market 2 is more similar to market 1 than market 3 for firm A, and vice versa for firm B). This example aims at capturing a situation where firms have different strongholds and their markets are the mirror image of each other. Although with a different formalization (which avoids the well-known continuity problems of the Hotelling model), this example describes a very similar situation as in a Hotelling model where two firms are located at the extremes of a line and consumers are distributed uniformly on that line. In such a model, if firm A is located at the extreme left and firm B at the extreme right of the line, firm A has a competitive advantage vis a vis those consumers that are located next to it (roughly corresponding to the group 1 of consumers in our example) and a competitive disadvantage vis a vis those located at the opposite extreme (corresponding to group 2 in our example), while the exact opposite holds for firm B. Consumers located in the middle of the centre (roughly corresponding to group 2 of consumers in our example) show a similar willingness to pay with respect to the two firms.

The way in which we proceed is as in the previous section. From the previous analysis it seems likely that - if it cannot fully price discriminate - firm A would like to set a common price in markets 1 and 2. We are then going to look at the best responses of firm B given firm A puts these two markets under the same price category.

Price categories $(12, 12)$ If firm B sets a common price in markets 1 and 2 as well, equilibrium prices can be easily derived by equations (45)-(46) and (39)-(40) above:

$$
\begin{aligned}
p^*_{12A}(12,12) &= \frac{1}{m+4}\left(v_1 + v_2 - \frac{m}{(3m+4)}(v_1 - v_3)\right) && (88)\\
p^*_{12B}(12,12) &= \frac{1}{m+4}\left(v_3 + v_2 - \frac{m}{(3m+4)}(v_3 - v_1)\right) && (89)
\end{aligned}
$$

$$
\begin{aligned}
p^*_{3A}(12,12) &= \frac{2}{m+4}\left(v_3 - \frac{m}{3m+4}(v_3 - v_1)\right) && (90)\\
p^*_{3B}(12,12) &= \frac{2}{m+4}\left(v_1 - \frac{m}{3m+4}(v_1 - v_3)\right) && (91)
\end{aligned}
$$

Profits in market 3 are:

$$\pi_{3A}^{*}(12,12) = \frac{(2+m)}{(m+4)^2(3m+4)^2}(2v_3(m+2)+v_1 m)^2 \tag{92}$$

$$\pi_{3B}^{*}(12,12) = \frac{(2+m)}{(m+4)^2(3m+4)^2}(2v_1(m+2)+v_3 m)^2 \tag{93}$$

and it is easy to check that they - not surprisingly - they are higher for firm B, which has a competitive advantage in market 3:

$$\pi_{3A}^{*}(12,12)-\pi_{3B}^{*}(12,12) = -\frac{(2+m)}{(m+4)(3m+4)}(v_1-v_3)(v_1+v_3)<0 \tag{94}$$

In the markets where firms set a common price, the equilibrium profits are:

$$\pi_{1A}^{*}(12,12)+\pi_{2A}^{*}(12,12) = \frac{1}{2}\frac{2+m}{(m+4)^2(3m+4)^2}(2(m+2)(v_1+v_2)+m(v_3+v_2))^2 \tag{95}$$

$$\pi_{1B}^{*}(12,12)+\pi_{2B}^{*}(12,12) = \frac{1}{2}\frac{2+m}{(m+4)^2(3m+4)^2}(2(m+2)(v_3+v_2)+m(v_1+v_2))^2 \tag{96}$$

Note that

$$\begin{aligned}
&[\pi_{1A}^{*}(12,12)+\pi_{2A}^{*}(12,12)]-[\pi_{1B}^{*}(12,12)+\pi_{2B}^{*}(12,12)] = \\
&\frac{1}{2}\frac{2+m}{(m+4)(3m+4)}\left((v_1+v_2)^2-(v_3+v_2)^2\right) = \\
&= \frac{1}{2}(m+2)(v_1-v_3)\frac{v_1+2v_2+v_3}{(m+4)(3m+4)}>0,
\end{aligned} \tag{97}$$

which implies that firm A makes larger profits in this pricing configuration than the rival: this is because the price categories better suit its market preferences than for its rival, which is setting a common price in two markets which are relatively dissimilar to each other. Total profits across all markets when both firms bundle $(1,2)$ together are thus:

$$\begin{aligned}
&\pi_{A}^{*}(12,12) = \\
&\frac{(2+m)}{(m+4)^2(3m+4)^2}\left((2(m+2)v_3+mv_1)^2+\frac{1}{2}(2(m+2)(v_1+v_2)+m(v_3+v_2))^2\right)
\end{aligned} \tag{98}$$

$$\begin{aligned}
&\pi_{B}^{*}(12,12) = \\
&\frac{(2+m)}{(m+4)^2(3m+4)^2}\left((2(m+2)v_1+mv_3)^2+\frac{1}{2}(2(m+2)(v_3+v_2)+m(v_1+v_2))^2\right)
\end{aligned} \tag{99}$$

Note that

$$\begin{aligned}
\pi_{A}^{*}(12,12)-\pi_{B}^{*}(12,12) &= \frac{(2+m)}{(m+4)(3m+4)}\left(\left(v_3^2-v_1^2\right)+\frac{1}{2}\left((v_1+v_2)^2-(v_3+v_2)^2\right)\right) \\
&= \frac{(2+m)}{2(m+4)(3m+4)}(v_1-v_3)(2v_2-v_1-v_3)
\end{aligned} \tag{100}$$

Hence, if $v_2 > \frac{v_1+v_3}{2}$,

$$\pi_{A}^{*}(12,12) > \pi_{B}^{*}(12,12) \tag{101}$$

Price categories $(12,23)$ Let us now see what happens when firm A sets a common price for markets 1 and 2 and firm B sets a common price for markets 2 and 3.

In the market 2, where firms set an independent price, equilibrium will be:

$$p_{3A}(12,23) = p_{1B}(12,23) = 2\frac{v_2 m + m v_1 + 3v_3 m + 8v_3}{(m+4)(5m+8)} \tag{102}$$

In the remaining markets:

$$p_{23B}(12,23) = p_{12A}(12,23) = 2\frac{2v_2 m + 2m v_1 + v_3 m + 4v_2 + 4v_1}{(m+4)(5m+8)} \tag{103}$$

Equilibrium profits are:

$$\begin{aligned} \pi_{3A}(12,23) = \pi_{1B}(12,23) = \\ = (v_2 m + m v_1 + 3v_3 m + 8v_3)^2 \frac{m+2}{(m+4)^2 (5m+8)^2} \end{aligned} \tag{104}$$

$$\begin{aligned} \pi_{2A}(12,23) = \pi_{2B}(12,23) \\ = -\left(-5v_2 m^2 - 24v_2 m - 24v_2 + 4m v_1 + 2v_3 m + 8v_1\right) \tfrac{2v_2 m + 2m v_1 + v_3 m + 4v_2 + 4v_1}{(m+4)^2 (5m+8)^2} \end{aligned} \tag{105}$$

$$\begin{aligned} \pi_{3B}(12,23) = \pi_{1A}(12,23) = \\ = \left(4m^2 v_1 + 20m v_1 + 24v_1 - v_2 m^2 - 8v_2 m + 2m^2 v_3 + 6v_3 m - 8v_2\right) \tfrac{2v_2 m + 2m v_1 + v_3 m + 4v_2 + 4v_1}{(m+4)^2 (5m+8)^2} \end{aligned} \tag{106}$$

Total profits per firm:

$$\begin{aligned} \pi_A(12,23) = \pi_B(12,23) = \\ = \tfrac{m+2}{(m+4)^2 (5m+8)^2} \left(2\left(2(m+2)(v_1+v_2) + v_3 m\right)^2 + \left(m(v_2+v_1) + v_3(3m+8)\right)^2\right) \end{aligned} \tag{107}$$

Price categories $(12,13)$ Let us now consider the case where firm A sets common price for markets 1 and 2, whereas firm B sets a common price for markets 1 and 3.

Equilibrium prices are:

$$\begin{aligned} p_{3A}(12,13) &= 2\tfrac{3m^2 v_1 + v_2 m^2 + 11m^2 v_3 + 8m v_1 + 56v_3 m + 64v_3}{(3m+8)(5m+8)(m+4)} \\ p_{2B}(12,13) &= 2\tfrac{64v_2 + 56v_2 m + 8m v_1 + 3m^2 v_1 + m^2 v_3 + 11v_2 m^2}{(3m+8)(5m+8)(m+4)} \\ p_{13B}(12,13) &= 2\tfrac{6m^2 v_1 + 2v_2 m^2 + 7m^2 v_3 + 28m v_1 + 32v_3 m + 32v_3 + 4v_2 m + 32v_1}{(3m+8)(5m+8)(m+4)} \\ P_{12A}(12,13) &= 2\tfrac{28m v_1 + 32v_2 m + 2m^2 v_3 + 7v_2 m^2 + 6m^2 v_1 + 4v_3 m + 32v_1 + 32v_2}{(3m+8)(5m+8)(m+4)}. \end{aligned} \tag{108}$$

Equilibrium profits are given by:

$$\pi_{1A}(12,13) =$$
$$\left(\left(15m^3+112m^2+264m+192\right)v_1-\left(5m^3+42m^2+96m+64\right)v_2+\left(5m^3+24m^2+24m\right)v_3\right)\times$$
$$\frac{28mv_1+32v_2m+2m^2v_3+7v_2m^2+6m^2v_1+4v_3m+32v_1+32v_2}{\left(3m+8\right)^2\left(5m+8\right)^2\left(m+4\right)^2} \tag{109}$$
$$\pi_{2A}(12,13) =$$
$$-\left(-\left(19m^3+134m^2+288m+192\right)v_2+\left(3m^3+32m^2+88m+64\right)v_1+\left(m^3+8m^2+8m\right)v_3\right)\times$$
$$\frac{28mv_1+32v_2m+2m^2v_3+7v_2m^2+6m^2v_1+4v_3m+32v_1+32v_2}{\left(3m+8\right)^2\left(5m+8\right)^2\left(m+4\right)^2} \tag{110}$$
$$\pi_{3A}(12,13) =$$
$$\left(\left(3m^2+8m\right)v_1+v_2m^2+\left(11m^2+56m+64\right)v_3\right)^2\frac{m+2}{\left(3m+8\right)^2\left(5m+8\right)^2\left(m+4\right)^2} \tag{111}$$

$$\pi_{1B}(12,13) =$$
$$-\left(-\left(10m^3+82m^2+224m+192\right)v_3+\left(12m^2+56m+64\right)v_1-\left(5m^3+24m^2+24m\right)v_2\right)\times$$
$$\frac{6m^2v_1+2v_2m^2+7m^2v_3+28mv_1+32v_3m+32v_3+4v_2m+32v_1}{\left(3m+8\right)^2\left(5m+8\right)^2\left(m+4\right)^2} \tag{112}$$
$$\pi_{2B}(12,13) =$$
$$\left(64v_2+56v_2m+8mv_1+3m^2v_1+m^2v_3+11v_2m^2\right)^2\frac{m+2}{\left(3m+8\right)^2\left(5m+8\right)^2\left(m+4\right)^2} \tag{113}$$
$$\pi_{3B}(12,13) =$$
$$\left(\left(12m^3+92m^2+232m+192\right)v_1-\left(v_2m^3+8m^2+8m\right)v_2+\left(4m^3+10m^2-32m-64\right)v_3\right)\times$$
$$\frac{6m^2v_1+2v_2m^2+7m^2v_3+28mv_1+32v_3m+32v_3+4v_2m+32v_1}{\left(3m+8\right)^2\left(5m+8\right)^2\left(m+4\right)^2}. \tag{114}$$

Note that in the configuration $(12,13)$ a possible natural deviation to the (candidate) *price* equilibrium just found comes to mind. Namely, that when the weak market of firm B (that is, market 1) is small enough, firm B might prefer simply to give up selling in that market, and focusing in the other two strong markets, 2 and 3. A form that such a deviation might take is - given the other firm's price choice - to set a price $p_{13B}^{dev}(12,13) > p_{13B}(12,13)$ which is optimal for the strong market 3, foregoing sales and profits in the small market 1. In fact, it turns out that such a deviation would not be profitable,[26] for the following reason: when firm A sets the prices for the price category (12), it sets the full discrimination

[26] Details available from the authors. Calculations are contained in the Mathematica file "12JulyUncovered-MarketsSimplifiedREV.nb".

price for its small market 3. Therefore, the (candidate) equilibrium price $p_{3A}(12,13)$ will be low, to reflect the lower valuation firm A faces in that market. The fact that market 3 is 'free' implies therefore that firm A prices aggressively in that market. In turn, this limits the profitability of the deviation strategy of firm B: firm B would like to forego sales in market 1 and charge higher prices in its high valuation market 3, but competition from firm A in that market limits the extent to which it can raise the price $p_{13B}^{dev}(12,13)$. To sum up, such a deviation would not be profitable, and the solutions identified above are the price equilibria for the configuration $(12,13)$.[27]

Lemma 1 *When firm A sets price category* (12)*, firm B's best response is to set price category* (23)*.*

Proof. To prove this lemma, one just needs to compare the profits obtained by firm B when A sets (12). (i) We show it is better to set price category (23) rather than (12). (ii) We show it is better to set (23) rather than (13). (We omit the proof that it is never optimal to choose price uniformity because this is evident.)

(i) This requires to prove that $\pi_B(12,23) > \pi_B(12,12)$. The resulting inequality - even after all possible simplifications - is a rather lengthy and complex expression to deal with. Numerical solutions show that it is always satisfied. As an illustration, we provide here a couple of examples that satisfy the assumption $v_2 > (v_1+v_3)/2$. For instance, by normalizing $v_1 = 1$, and fixing $v_2 = 3/4$ and $v_3 = 1/4$, we have:

$$\pi_B(12,23) - \pi_B(12,12) = \frac{6144 + 19456m + 21888m^2 + 11168m^3 + 2610m^4 + 225m^5}{32(4+m)^2(4+3m)^2(8+5m)^2} > 0, \tag{115}$$

and for $v_1 = 1$, $v_2 = 3/4$ and $v_3 = 1/8$, we have:

$$\pi_B(12,23) - \pi_B(12,12) = \frac{3\left(3584 + 10880m + 11960m^2 + 6020m^3 + 1394m^4 + 119m^5\right)}{32(4+m)^2(4+3m)^2(8+5m)^2} > 0, \tag{116}$$

(ii) This requires to prove that $\pi_B(12,23) > \pi_B(12,13)$. Fortunately, this inequality turns out to be much simpler to study. One can check that:

$$\pi_B(12,23) - \pi_B(12,13) = \frac{16\,(2+m)^2\left[(8+3m)\,v_1 - (4+3m)\,v_2 - 4v_3\right](v_2 - v_3)}{(8+3m)^2(8+5m)^2} > 0. \tag{117}$$

[27] Obviously, the same holds for the configuration $(13,23)$, which is the mirror image of $(12,13)$.

Note that the numerator is positive by the assumption that $v_1 \geq v_2 \geq v_3$. ■

We can now note that our problem is symmetric, in the sense that market 2 is identical for the two firms and that markets 1 and 3 are just inverted for one firm relative to the other. Therefore, we can establish the following proposition:

Proposition 3 *The price categories configuration* $(12, 23)$ *is an equilibrium of the full game when firms are asymmetric (in the sense that* $v_{1A} = v_{3B}$ *and* $v_{3A} = v_{1B}$*).*

Proof. We have already proved that (23) is B's best reply to A setting (12). We now need to prove that (12) is A's best reply when B sets the price category (23). To do so, we have to show: (i) that $\pi_A(12, 23) > \pi_A(23, 23)$, and (ii) that $\pi_A(12, 23) > \pi_A(13, 23)$.

(i) By the symmetry of the problem, that is given the hypotheses made on the willingness to pay in the different markets, the results obtained for firm B can be used for firm A. In particular, $\pi_A^*(23, 23) = \pi_B^*(12, 12)$, and $\pi_A^*(12, 23) = \pi_B^*(12, 23)$. Therefore, from the result proved above that $\pi_B^*(12, 23) > \pi_B^*(12, 12)$ it follows that $\pi_A(12, 23) > \pi_A(23, 23)$.

(ii) Again by the symmetry of the problem, we have, $\pi_A^*(23, 23) = \pi_B^*(12, 12)$, and $\pi_A^*(13, 23) = \pi_B^*(12, 13)$. Therefore, from the result proved above that $\pi_B^*(12, 23) > \pi_B^*(12, 13)$ it follows that $\pi_A(12, 23) > \pi_A(13, 23)$. ■

After having found that $(12, 23)$ is an equilibrium, we now show that it is the unique equilibrium.

Proposition 4 *The price categories configuration* $(12, 23)$ *is the only equilibrium of the full game when firms are asymmetric (in the sense that* $v_{1A} = v_{3B}$ *and* $v_{3A} = v_{1B}$*).*

Proof. Most of the possible candidate equilibria of the whole game can already be excluded due to the profit comparisons made above. However, we should still prove that $(13, 13)$ is not an equilibrium of the game. First of all, let us find the equilibrium profits under this price configuration. Equilibrium prices are:

$$p_{2A}^*(13, 13) = p_{2B}^*(13, 13) = \frac{2v_2}{m+4}; \tag{118}$$

$$p_{1A}^*(13, 13) = p_{3A}^*(13, 13) = p_{1B}^*(13, 13) = p_{3B}^*(13, 13) = \frac{v_1 + v_3}{m+4} \tag{119}$$

As for profits they will be:

$$\pi_A^*(13, 13) = \pi_B^*(13, 13) = \frac{1}{2}\frac{2+m}{(m+4)^2}\left(2v_2^2 + (v_1 + v_3)^2\right). \tag{120}$$

Note that in this case the results are perfectly symmetric because in this pricing configuration each firm is setting a common price in the highest and lowest markets.

To exclude that this configuration is an equilibrium, it is sufficient to show that a firm has an incentive to deviate from it. One such a deviation is the following: when B is setting a common price (13), firm A prefers to set a price category (12) rather than the price category (13). Therefore, it is sufficient to prove that $\pi_A^*(12,13) > \pi_A^*(13,13)$ to see that the candidate equilibrium is broken. We have carried out numerical solutions and showed that $\pi_A^*(12,13) > \pi_A^*(13,13)$. As an illustration, let us consider two examples that satisfy the assumption $v_2 > (v_1+v_3)/2$. By normalizing $v_1 = 1$, and fixing $v_2 = 3/4$ and $v_3 = 1/4$, we have:

$$\pi_A(12,13) - \pi_A(13,13) = \frac{(2+m)\left(32768 + 50176m + 27456m^2 + 6304m^3 + 513m^4\right)}{32(4+m)^2(8+3m)^2(8+5m)^2} > 0, \tag{121}$$

and for $v_1 = 1$, $v_2 = 3/4$ and $v_3 = 1/8$, we have:

$$\pi_A(12,13) - \pi_A(13,13) = \frac{3\,(2+m)\left(61440 + 94208m + 51932m^2 + 11680m^3 + 931m^4\right)}{128(4+m)^2(8+3m)^2(8+5m)^2} > 0. \tag{122}$$

Finally, one might wonder if there might be a pricing equilibrium for the configuration $(13,13)$ where each firm sets a higher price than $p_{13S}(13,13)$ (with $S = A, B$) and just sells in its high valuation market. That is, firm A would set the price p_{13A} which corresponds to the monopoly price for market 1 and does not sell in market 3, while firm B would set the price p_{13B} corresponding to the monopoly price for market 3 and does not sell in market 1. This strategy would indeed give higher profits to the firms than the profits arising under the other equilibria, since each firm would monopolize its captive market, and competition would be left only for the common market 2. However, it turns out that such pricing strategies would not form an equilibrium. (Proof available from the authors. Mathematica file "12July2004UncoveredMarketsSimplifiedREV.nb".) Indeed, given the monopoly price set by, say, firm B in market 3, firm A would have an incentive to deviate and choose a lower price at which it has positive sales in market 3 (while retaining a monopoly in market 1).

$\blacksquare$

6.2 Dynamic model (repeated market interaction)

So far, we have analyzed a static game, where firms meet in the marketplace only once. We now study a dynamic model, where firms choose in the beginning of the game whether to use price categories (and if so, which ones), and they then choose prices in each period for an infinite number of periods. This setting will allow us to check whether price categories choices might have an effect on the likelihood of collusion in the industry. (Collusion can arise only in games with infinite horizon, or - which is equivalent - in games with uncertain finite date, see Motta (2004: chapter 4) for a discussion of collusion.)

Notice that throughout this Section we shall assume that the decision taken about price categories in the beginning of the game represent a commitment for the firms. If this were not the case, that is if firms could change pricing policies with respect to categories in any period, then there would be no effect on collusion.

We also focus on collusive strategies which entail Nash reversal forever along the punishment path (that is, after a deviation, firms go back to the one-shot Nash equilibrium of the respective price category, that we have already identified in the previous Section.)

Since we have already seen in the previous Section that in a static game firms would choose the same price categories in a symmetric environment, we focus on the asymmetric example of the previous section. In other words, the objective of our Report is to study what might push firms to choose the same price categories. We know that in a static environment with symmetric market valuations there is no need to resort to collusive motives to explain the same choice of price categories; we now want to see if such a collusive rationale might exist when asymmetric valuations exist.

6.2.1 The model

The model analyzed in this Section is the same as Example (ii) in the previous Section. We assume $v_{1A} = v_{3B} = v_1$, $v_{3A} = v_{1B} = v_3$, and $v_{2A} = v_{2B} = v_2$, with $v_1 > v_2 > v_3$, and $v_2 > (v_1 + v_3)/2$ (that is, market 2 is more similar to market 1 than market 3 for firm A, and vice versa for firm B). For simplicity, in this Section we also add the assumption $v_3 < v_1/2$.[28]

We also know that in this asymmetric (but mirror images) context, the equilibrium choice of price categories in the static game would be $(12, 23)$: each firm would set a common price in

[28] This assumption serves to avoid slightly more complex collusive schemes where a firm receives a positive demand but decides not to supply it. In other words, this assumption allows us to consider in this Section only collusive outcomes where firms decide prices, but not quotas. (Fixing also quotas, or market shares, being a more complex collusive scheme that is more difficult to implement and probably easier to detect in practice.)

those markets which are more similar to each other, namely 1 and 2 for firm A and 2 and 3 for firm B. In what follows, though, we are going to show that if interaction in the marketplace is repeated infinitely, firms might have an incentive to set the same price category $(13, 13)$ as a way to facilitate collusion. The intuition for this result is that by setting a common price in its most important market and in its smallest market, a firm has an incentive to be less aggressive in pricing. Reducing the common price to achieve higher market shares in the less important market would be at the cost of losing profits on the most important market. In a collusive equilibrium, each firm sets the common price at the level which maximizes its monopoly profit in its biggest market and does not sell in its low valuation market. So, each firm becomes a monopolist in its captive market (remember, like in a Hotelling model here the most important market for one firm is the least important for the other, and vice versa). (In the middle market, the usual collusive price applies.) This collusive pricing strategy (that we might call of 'local monopolies' thinking of the similarity with the Hotelling model), with each firm serving its 'captive' market, could not arise at the equilibrium of the static game, since - given the strategy of the rival - in a one-shot price game a firm has an incentive to slightly decrease the common price so as to gain a positive market share in the low valuation market (while keeping a monopoly in the highest valuation market). But in a repeated price game, such a deviation would be met by a punishment of the rival, and could therefore be sustained for high enough discount factors.

To be more precise, the sustainability of collusive outcomes increases with (i) the collusive profits and decreases with (ii) the deviation profits and (iii) the profits along the punishment path. In what follows we show that choosing the same price categories $(13, 13)$ gives rise to higher collusive profits, lower deviation profits and lower punishment profits than the price category $(12, 23)$. Therefore, the former strategy unambiguously facilitates collusion: there exists an interval of the discount factor where collusion could be sustained if firms chose the same price category, but could not be sustained if firms chose different price categories.

Call π_S^c the profits that firm S receives if it chooses a certain collusive action, given that all firms also collude. Call π_S^d the profit of firm S if it deviates when the rival takes the collusive action, and π_S^p the value of firm S's profits in the punishment phase, that is in all periods that follow the deviation period. Denote with $\delta \in (0, 1)$ the discount factor, assumed identical for both firms in the industry. Note that $\delta = 0$ corresponds to the case where people are infinitely impatient, and do not attach any value to the future, and that $\delta = 1$ corresponds to the case where people are infinitely patient, and they attach equal value to the future as to the present.

Collusion can arise only if each firm will prefer to play the collusive action rather than deviate from it (and be punished). Therefore, it must be that the following *Incentive Constraints* (ICs) holds:

$$\frac{\pi_S^c}{1-\delta} \geq \pi_S^d + \frac{\delta \pi_S^p}{1-\delta} \qquad S = A, B. \tag{123}$$

Clearly, the lower the deviation profit one makes relative to the collusive profit, and the lower the profit in the punishment phase, the more likely that collusion will be sustained. (The harsher the punishment the stronger the deterrent to cheating on the collusive agreement.) The incentive constraints can also be written as:

$$\delta \geq \frac{\pi_S^d - \pi_S^c}{\pi_S^d - \pi_S^p} \equiv \overline{\delta}_S \qquad S = A, B. \tag{124}$$

Collusion arises at equilibrium only if the discount factor is large enough, that is, if it is larger than the "critical discount factor", $\overline{\delta}$. In what follows, we shall have to identify and compare the critical discount factors in the cases of price categories $(13, 13)$ and $(12, 23)$. For each case, this will amount to finding the values of the profits π_S^c, π_S^d and π_S^p (the latter being known from the analysis of the previous Section: punishment profits equal the one-shot equilibrium profits under the collusive strategies with Nash reversal that we consider here).

6.2.2 Collusive profits

The highest collusive profits that firms can obtain in this industry coincide with the profits achieved if the firms jointly maximized their profits. If the two firms were jointly owned, and they did not have pricing restrictions (i.e., if they could fully price discriminate), the joint-profit maximizing solution would be given by:

$$p_{1A}^c = p_{3B}^c = p_{3A}^c = p_{1B}^c = v_1/2; \quad p_{2A}^c = p_{2B}^c = v_2/2. \tag{125}$$

At these prices, firm A would be the sole seller in market A and firm B would be the sole seller in market 3, whereas the two firms could share market 2 equally. Each firm would have collusive profits:

$$\pi_S^c = \frac{2v_1^2 + v_2^2}{8}, \qquad S = A, B \tag{126}$$

Indeed, it is easy to check that at the above mentioned prices we would have $q_{3A} = q_{1B} = \frac{1}{2}(v_3 - v_1/2) < 0$: namely, firms A and B will have zero output in their least important market at the collusive outcome. Further, one can check that having firms A also serving market 3 and firm B serving market 1 would be suboptimal. By maximizing

joint profits $\pi = \sum_{i=1,2,3} p_{iA} q_{iA}(p_{iA}, p_{iB}) + \sum_{i=1,2,3} p_{iB} q_{iB}(p_{iA}, p_{iB})$ where $q_{iA}(p_{iA}, p_{iB}) > 0$ and $q_{iB}(p_{iA}, p_{iB}) > 0$ are given by equations (1) would result in prices $p_{1A} = p_{3B} = \frac{(2+m)v_1 + mv_3}{4(1+m)}$, $p_{3A} = p_{1B} = \frac{(2+m)v_3 + mv_1}{4(1+m)}$ and $p_{2A} = p_{2B} = v_2/2$. Profits would be $\pi_S = \frac{(2+m)(v_1^2+v_3^2)+2(1+m)v_2^2+2mv_1v_3}{16(1+m)} < \pi_S^c$.[29] Therefore, a situation where each firm sells in its captive market would give higher profits to the firms.

6.2.3 Conditions for collusion: configuration $(13, 13)$

Collusive profits - for configuration $(13, 13)$ Once identified the highest collusive profits under full discrimination, note that the price category $(13, 13)$ would allow firms to reproduce this outcome. Each firm would set the common price $v_1/2$ for the highest and the lowest valuation market, and sell only in the former, while sharing the market with the rival in market 2, where firms would charge the joint-profit maximizing price v_2. Each firm would therefore set

$$p_{13A}^c(13,13) = p_{13B}^c(13,13) = v_1/2; \quad p_{2A}^c(13,13) = p_{2B}^c(13,13) = v_2/2. \tag{127}$$

and earn:

$$\pi_A^c(13,13) = \pi_B^c(13,13) = \frac{2v_1^2 + v_2^2}{8}. \tag{128}$$

Note that the highest collusive outcome cannot be reproduced in the price configuration $(12, 23)$ - or even in configurations $(12, 12)$, $(23, 23)$: this is because the optimum requires two different prices for market 1 and for market 2, something which cannot be done if firms set common prices in markets 1 and 2 or 2 and 3. We shall come back to the collusive outcome for $(12, 23)$ below, but before that, let us find the other profit variables which are crucial to identify the critical discount factor for the configuration $(13, 13)$.

Deviation profits - for configuration $(13, 13)$ We now have to find the optimal deviation profits for a firm when both firms have committed to price categories $(13, 13)$ and given that the rival plays the collusive strategy given by equation (125) above. Given that B plays $p_{1B}^c = p_{3B}^c = v_1/2$ and $p_{2B}^c = v_2/2$, firm A will want to choose the price at which firm B's demand will be zero (a lower price would decrease profits),[30] unless the products are

[29] Since $\pi_S^c - \pi_S = \frac{(v_1 - v_3)((2+3m)v_1 + (2+m)v_3)}{16(1+m)} > 0$.

[30] To be more precise, with the demand function adopted here, firm A will want firm B to sell an arbitrarily small quantity $\epsilon > 0$. This is because the utility function from which demands are derived exhibits a positive externality in the valuation of the other good.

almost independent. In what follows, we restrict the analysis to $m > 5$ to exclude optimal deviations which leave the rival with positive output.[31] By setting $q_{2B}(p_{2A}, v_2/2) = 0$ and $q_{3B}(p_{13A}, v_1/2) = 0$ one obtains the deviation prices:

$$p^d_{13A}(13,13) = \frac{(m-2)v_1}{2m}; \quad p^d_{2A}(13,13) = \frac{(m-2)v_2}{2m}, \tag{129}$$

and deviation profits

$$\pi^d_A(13,13) = \frac{(m-2)\left((3+m)\,v_1^2 + (1+m)\,v_2^2 + mv_1v_3\right)}{8}. \tag{130}$$

Note also that this deviation strategy makes sense only if $m > 2$, that is if products are not extremely differentiated.

Punishment profits - for configuration $(13,13)$ After a deviation, our collusive strategies require firms to go back to the one-shot Nash equilibrium forever. We have already computed such profits in the previous Section, and they are:

$$\pi^p_A(13,13) = \frac{(m+2)\left(v_1^2 + 2v_2^2 + 2v_1v_3 + v_3^2\right)}{2(4+m)^2}. \tag{131}$$

We can derive the critical discount factor under this product configuration as:

$$\overline{\delta}_S(13,13) \geq \frac{\pi^d_S(13,13) - \pi^c_S(13,13)}{\pi^d_S(13,13) - \pi^p_S(13,13)} =$$
$$\frac{2(m-6)v_1^2 + (m^2 - 2m - 4)v_2^2 + 2m(m-2)v_1v_3}{2m^2\left(\frac{(m-2)\left((3+m)v_1^2+(1+m)v_2^2+mv_1v_3\right)}{m^2} - \frac{2(m+2)\left(v_1^2+2v_2^2+2v_1v_3+v_3^2\right)}{(4+m)^2}\right)}, \quad S = A, B. \tag{132}$$

Figure 2 draws the critical discount factor $\overline{\delta}_S(13,13)$ for values $v_1 = 1$, $v_2 = 3/4$ and $v_3 = 1/4$, which satisfy our assumptions ($v_1 = 1$ is just a normalization). Note that the figure is drawn for $m > 5$ (for lower values there are possible discontinuities).

6.2.4 Condition for collusion: configuration (12,23)

We now turn to the identification of the conditions for collusion - i.e., the critical discount factor - in the price configuration $(12,23)$. We first identify the collusive profits, then the profits under deviation and along the punishment path.

[31] The assumption that $m > 5$ is made for simplicity, in order to avoid consideration of other sub-cases. Note also that this amounts to disregarding the uninteresting case where products are almost independent.

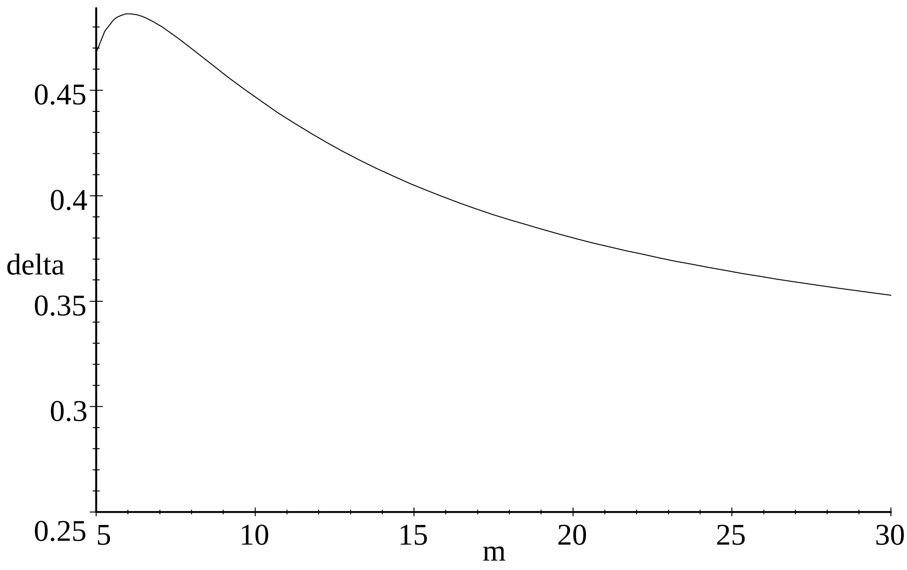

Figure 2: *Critical discount factor for configuration (13,13)*

Collusive profits - for configuration $(12, 23)$ We have seen above that the highest collusive profits that firms can obtain in this industry is achieved when firms can implement prices $v_1/2$ in markets 1 and 3, and price $v_2/2$ in market 2, with the firms sharing market 2 and selling only in their highest valuation market. However, such prices cannot be implemented (unless perhaps if they resorted to complicated output allocation rules as well) if firm A has to set a common price in markets 1 and 2 and firm B a common price in markets 2 and 3. Joint-profit maximisation results instead in the following collusive prices:

$$p_{12A}^c(12,23) = p_{23B}^c(12,23) = p_{3A}^c(12,23) = p_{1B}^c(12,23) = \frac{2v_1 + v_2}{6}. \tag{133}$$

At these prices, firm A (resp. B) would sell in market 1 (resp. 3) as a monopolist and in market 2 sharing the market with the rival.[32] Collusive profits will be:

$$\pi_A^c(12,23) = \pi_B^c(12,23) = \frac{(2v_1 + v_2)^2}{24}. \tag{134}$$

Deviation profits - for configuration $(12, 23)$ We now have to find the optimal deviation profits for a firm when both firms have committed to price categories $(12, 23)$

[32] One can check that $q_{3A} = q_{1B} < 0$ if $v_3 < \frac{1}{6}(2v_1 + v_2)$. In other words, firms A and B will have zero output at the collusive outcome in their respective lowest valuation markets, as long as v_3 is small enough. In what follows, we assume that this restriction is satisfied.

and given that the rival plays the collusive strategy given by equation (133) above. Given that B plays $p^c_{23B}(12,23) = p^c_{1B}(12,23) = \frac{2v_1+v_2}{6}$, firm A will want to choose the price at which firm B's demand will be zero (a lower price would decrease margins without increasing demand), under our assumption that products are not extremely differentiated (recall that for simplicity we have assumed in this Section that $m > 5$). By setting $q_{2B}(p_{12A}, \frac{2v_1+v_2}{6}) = 0$ and $q_{3B}(p_{3A}, \frac{2v_1+v_2}{6}) = 0$ one obtains firm A's deviation prices as:

$$p^d_{12A}(12,23) = \frac{2(m+2)v_1 + (m-10)v_2}{6m}; \quad p^d_{3A}(12,23) = \frac{2(m-4)v_1 + (m+2)v_2}{6m}, \quad (135)$$

and the associated deviation profits will be:

$$\pi^d_A(12,23) = \frac{\left(6m^2-4m-56\right)v_1^2+\left(9m^2+32m+136\right)v_1v_2+6m(m-4)v_3+\left(\left(3m^2-28m-152\right)v_2+3m(2+m)v_3\right)v_2}{36m^2}. \quad (136)$$

Punishment profits - for configuration $(12,23)$ After a deviation, our collusive strategies require firms to go back to the one-shot Nash equilibrium forever. We have already computed such profits in the previous Section, and they are:

$$\pi^p_A(12,23) = \frac{m+2}{(m+4)^2\,(5m+8)^2} \times \quad (137)$$

$$\left(2\left(2\left(m+2\right)\left(v_1+v_2\right)+v_3 m\right)^2 + \left(m\left(v_2+v_1\right)+v_3\left(3m+8\right)\right)^2\right). \quad (138)$$

We can derive the critical discount factor under this product configuration as:

$$\overline{\delta}_S(12,23) \geq \frac{\pi^d_S(12,23) - \pi^c_S(12,23)}{\pi^d_S(12,23) - \pi^p_S(12,23)}, \quad (139)$$

whose expression we omit for space.

Figure 3 draws the critical discount factor $\overline{\delta}_S(12,23)$ for values $v_1 = 1$, $v_2 = 3/4$ and $v_3 = 1/4$, which satisfy our assumptions ($v_1 = 1$ is just a normalization). Note that the figure is drawn for $m > 5$ (for lower values there are possible discontinuities).

6.2.5 Conditions for collusion: comparison

Now that we have obtained the collusion, deviation and punishment payoffs for the firms, and obtained the critical discount factors, in the two alternative configurations $(13,13)$ and

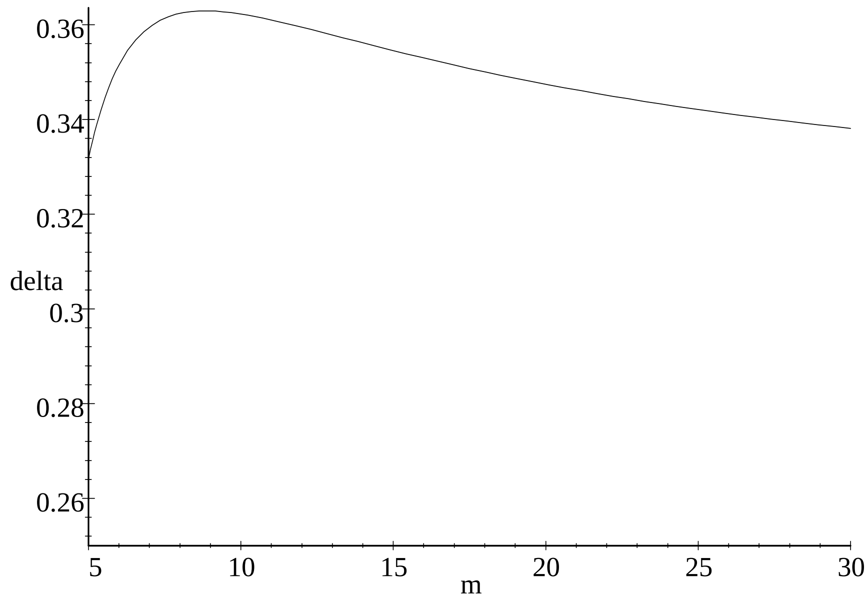

Figure 3: *Critical discount factor for configuration (12,23)*

$(12, 23)$, we can compare them. We show here that choosing the same price categories $(13, 13)$ will favor the sustainability of collusion.

First of all, note that the configuration where both firms choose the same price category $(13, 13)$ will give the firms (weakly) higher collusive profits than the configuration $(12, 23)$ which is the equilibrium in the static game (the collusive profits under the two different price configurations will coincide in the particular case where $v_1 = v_2$):

$$\pi_S^c(13, 13) - \pi_S^c(12, 23) = \frac{(v_1 - v_2)^2}{12} \geq 0. \tag{140}$$

This result does not surprise us, as it also follows from the fact that under $(13, 13)$ it is possible to achieve the highest collusive profits, the same as firms could attain under collusion and full price discrimination.

Next, we compare the deviation profits obtained under the two different price categories, as:

$$\pi_S^d(12, 23) - \pi_S^d(13, 13) = \frac{(v_1 - v_2)\left[-\left(3m^2 + 13m + 2\right)v_1 + \left(6m^2 + 19m + 134\right)v_2 - (3m^2 + 6m)v_3\right]}{36m^2} > 0, \tag{141}$$

the inequality being positive due to our assumption that $v_2 > (v_1 + v_3)/2$.

To understand why the deviation profits are higher under categories $(12, 23)$ is not so immediate here, as the collusive prices themselves are different under the two configurations. A possible intuition for this result is that under category pair $(12, 23)$ the price which is independent is the one which corresponds to the weakest market for the deviating firm, and the strongest market for the rival (this the market where the gains from deviation are higher, as under collusion sales for the firm which is considering deviation are nil). The deviating firm can be more aggressive in that market than in the case $(13, 13)$ where it sets a common price in the weakest and strongest market: in the latter, a price decrease in the weal market decreases the payoffs from the strongest market, where it is a monopolist.

Finally, we compare the punishment profits, as follows:

$$\pi_A^p(12,23) - \pi_A^p(13,13) =$$
$$-\tfrac{(2+m)}{2(4+m)^2(8+5m)^2}[-64(2v_1 - v_2 - v_3)(v_2 - v_3) + 16m(v_1^2 - 8v_1v_2 + 6v_2^2 + 6v_1v_3 - 4v_2v_3 - v_3^2)] + \tag{142}$$
$$-\tfrac{(2+m)}{2(4+m)^2(8+5m)^2}[m^2(7v_1^2 - 36v_1v_2 + 32v_2^2 + 22v_1v_3 - 28v_2v_3 + 3v_3^2)] > 0$$

This result follows from the discussion of the static game. Indeed, we know that under price categories $(12, 23)$ each firm sets a common price in the two markets which are more similar to each other, which therefore implies less pricing distortion from the full discrimination case than in the case where the common price was set in the less similar price categories $(13, 13)$.

From the results obtained for these comparisons, it follows unambiguously that collusion is more easily sustained under the configuration $(13, 13)$. Indeed, we know that in a given configuration collusion can be sustained if the following incentive constraints are satisfied for each firm $S = A, B$: $\frac{\pi_S^c}{1-\delta} \geq \pi_S^d + \frac{\delta \pi_S^p}{1-\delta}$. Since $\pi_S^c(13, 13) > \pi_S^c(12, 23)$, $\pi_S^d(13, 13) < \pi_S^d(12, 23)$, and $\pi_S^p(13, 13) < \pi_S^p(12, 23)$, it follows that the ICs are slacker under the configuration $(13, 13)$.

The following Figure 4 compares the two critical discount factors, and illustrates our result that $\overline{\delta}_S(12, 23) > \overline{\delta}_S(13, 13)$, that is, collusion is facilitated when firms choose the same price configuration $(13, 13)$.

6.3 Semi-Collusion

So far, our technical treatment has considered firms that were constrained to offer price categories for exogenous reasons. However, price categories could also emerge *endogenously*, as a

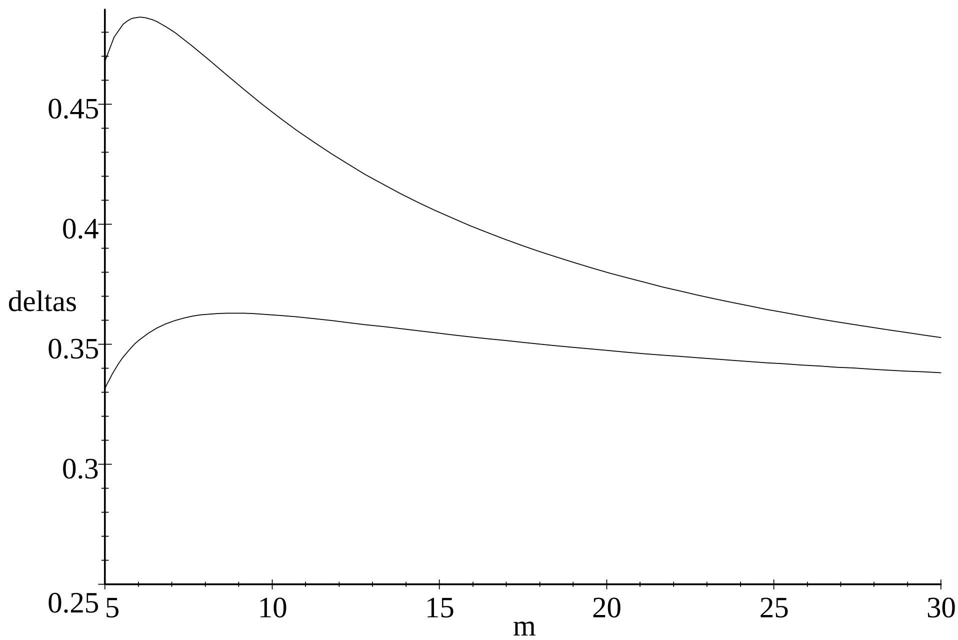

Figure 4: *Critical discount factors. The solid line refers to configuration (13,13) and the dotted line to configuration (12,23)*

result of firms' coordination. This possibility raises the question of whether anti-competitive reasons might explain why firms choose to offer price categories in the first place. Stated differently, would firms ever want to commit to price categories? The answer to this question is not obvious. Committing to offering price categories restrict firms' behavior. In particular, firms cannot price discriminate any more in the markets that are included in a price category. On the other hand, it is well-known that joint commitment can bring benefits in non-cooperative games.

To repeat, the main difference between this analysis and the one presented earlier is that, while the previous analysis assumes that price categories have to be used for *exogenous* reasons and focuses on their design, we consider here a semi-collusion game and ask whether price categories can emerge *endogenously*. To address this question, we investigate a semi-collusion game where firms can collude on price categories but not on prices. We use the same setup as the one used earlier but focus on a static game with symmetric firms. This restricted setup is sufficient to answer the question asked. Also, note that the analysis of this Section is not aimed at addressing the question of why firms might want to choose the *same* price categories, but if they might want to choose price categories at all.

Under semi-collusion, firms constrain themselves to charge the same price in a given set of markets. We ask whether firms can be better-off under such a commitment. We demonstrate that for some particular markets, the answer is affirmative. The basic intuition is that by committing to uniform pricing in a subset of markets, firms can soften the impact of price competition. This suggests that firms may use price categories as an alternative collusion device when they cannot directly collude on prices.

We show, however, that firms will include in a price category only markets that differ according to their level of competition. To illustrate the intuition for this result, consider a simple example where firms are symmetric and compete in two markets that are identical in all dimensions but differ in their levels of competition. Stated differently, firms' products are closer substitutes in one market than in the other. Although the level of competition differs across markets, the sum of the firms' demand is the same in both markets. As a consequence, a monopolist controlling all products, would set the same price for both firms' products (because of the firm symmetry assumption) in both markets (because there is no reason to charge different prices in markets that have identical demands).

Consider firm behavior under semi-collusion. If firms do not commit to price categories, prices will vary in equilibrium across markets. Prices will be lower in the more competitive market. Varying prices, however, lowers joint profits because total market demands are

identical by assumption. We show that for markets with low levels of market competition, firms are better-off committing to a single price. We present examples where price categories emerge *endogenously* as the unique semi-collusion outcome.

A possible implication of the analysis is that firms may want to collude on price categories when they cannot collude on price. The welfare implications of price categories are ambiguous. Under price categories, the price increases in more competitive markets and decreases in less competitive ones. We cannot say anything in general on the impact of price categories on the overall level of prices. This impact depends on the demand specification.

The framework We focus on a duopoly situation and consider a two-stage semi-collusion game where firms can collude on price categories but not on prices. We use the same demand specification used in the previous analysis but focus on a static game with symmetric firms. The reason for this restricted setup it is sufficient to answer the question asked.

Because we consider symmetric firms, we restrict to symmetric equilibria in both stages. In the first stage, firms commit to the same price categories. In the second stage firms set the same prices in equilibrium.

We restrict to semi-collusion games to focus on the choice of price categories. From a single firm's point of view, it is always optimal to choose price discrimination in the first stage because price categories reduces the firm's degree of freedom in the second stage. Therefore, price categories would not emerge (excluding trivial cases such as identical markets) if firms would compete in both stages.

We want to understand the characteristics of markets that influence the decision to commit to price categories. The price category decision is potentially complex. If there are 3 markets, firms can choose between 3 categories, in addition to price discrimination and uniform pricing, in the first stage. If there are 4 markets, firms face of choice set of 14 different price categories (6 two-products, 4 three-products, and 4 pairs of two-products). As the number of markets increases, the choice set increases exponentially. Since our main concern is to demonstrate the possibility of price categories and to understand what characteristics of markets determine the choice of price categories, our analysis leaves aside the most general treatment of the problem.

Our analysis proceeds in two steps. We first analyze the simple benchmark case where firms compete in only two markets. In that situation, the duopolists have to choose in the first stage between uniform pricing and price discrimination. This could be qualified as a degenerated case since there is no possibility for price categories as defined above. As it

turns out, however, this simple benchmark case is an essential building block to understand when price categories emerge under semi-collusion when there are more than 2 markets. This shall be the focus of the second step of the analysis.

Some notation will help to explain the argument presented in the second step. Assume a market is characterized by a vector $M \in \mathbf{M}$, where $\mathbf{M}$ is the population of all possible markets where firms may compete. When firms set uniform price in (M_1, M_2), we call the resulting market $M_{1,2}$. Assume for now that $M_{1,2}$ is also an element of $\mathbf{M}$ and that market aggregation is independent of the order of aggregation $M_{1,(2,3)} = M_{(1,2),3}$, and denote the resulting market $M_{1,2,3}$. (These assumptions hold for our demand specification.) Assume we have characterized the first step decision for any pair $(M_1, M_2) \in \mathbf{M}x\mathbf{M}$ as follows: If firms prefer price discrimination over uniform pricing for market M_1 and M_2 we say that $(M_1, M_2) \succ M_{1,2}$. If uniform pricing dominates price discrimination, then $M_{1,2} \succ (M_1, M_2)$. If we can show that there exists three markets M_1, M_2 and M_3 such that $M_{1,2} \succ (M_1, M_2)$ and $(M_{1,2}, M_3) \succ M_{1,2,3}$, then this would implies that price categories will emerge in equilibrium. In fact, the first relation says that price discrimination cannot be an equilibrium while the second says that uniform pricing cannot be an equilibrium.

The rest of this note is organized as follows. The next section presents the model. The following section considers the two market case. In the final section, we show that price categories can emerge under semi-collusion and discuss the characteristics of markets where price categories are likely to emerge.

6.3.1 The Model

We use a model very similar as the one used earlier. Because the questions we pose address new issues, however, the models differ on a few points. The main difference is that we consider markets that differ on more dimensions than just willingness to pay but we focus on firms that are symmetric in each market. For the sake of clarity, we expose all features of the model. This is done at the cost of possible repetitions but it is worthwhile to warrant that the analysis is self-contained.

We consider two firms A and B which sell in multiple markets $i = 1..I$. Market i is characterized by a four dimensional vector $M_i = (v_i, c_i, m_i, s_i)$ where v_i captures the willingness to pay in market i, c_i the marginal cost of production in market i, m_i the level of market competition in market i, and s_i the size of market i. The demand function faced by firm A in market M_i is:

$$q_{i,A} = \frac{1}{2} s_i \left(v_i - p_{i,A} \left(1 + \frac{m_i}{2} \right) + \frac{m_i}{2} p_{i,B} \right) \tag{143}$$

The demand faced by firm B is defined similarly. The level of market competitiveness measures how substitutable the firms' products are in each market. The profits of firm A in market M_i are:

$$\pi_{i,A} = \frac{1}{2}s_i\left(v_i - p_{i,A}\left(1 + \frac{m_i}{2}\right) + \frac{m_i}{2}p_{iB}\right)(p_{i,A} - c_i) \tag{144}$$

The profits of firm B are defined similarly. The two stage game was defined in the previous section. In the second stage, we assume that firms compete in price. Firms are symmetric so we restrict to symmetric equilibrium in pure strategy in both stages of the game.

We present the market equilibrium when firms compete in a single market because all the derivations that follow rely on these results. Consider a single market $M = (v, c, m, s)$. The first stage decision is trivial since firms face a single market. The equilibrium price and profits in the second stage are

$$p = \frac{2\left(v + c\left(1 + \frac{m}{2}\right)\right)}{m + 4} \tag{145}$$

$$\pi = (v - c)^2 \frac{2 + m}{(m + 4)^2} \tag{146}$$

See Motta (2004) subsection 8.4.2.2 for a derivation of these results.

6.3.2 Price Discrimination versus Uniform Pricing

In this section, we assume that firms A and B sell only in two markets $M_1$and M_2 Since the two markets vary only along four dimensions, we restrict to four simple benchmark cases. We consider in turn pairs of markets that differ on one of the 4 dimensions of M. We ask whether profits are higher under price discrimination than under uniform pricing. Although less general than considering any pair of markets, this approach has the advantage of bringing to light the intuition that determine the firms' choice of pricing strategies in the first stage. In addition, these simple benchmark cases are essential to demonstrate that price categories can emerge in equilibrium, which is the aim of the next section.

Throughout this section, we use the superscripts U and D to denote uniform pricing and price discrimination respectively. For example, p_i^D is the equilibrium price in market i under price discrimination. Under uniform pricing, we impose

$$p_1^U = p_2^U \tag{147}$$

We denote π^X a firm's profits across both market for $X = D, U$

$$\pi^X = \sum_{i=1,2} \pi_i^X \tag{148}$$

Willingness to Pay This subsection follows the line of the analysis presented in the analysis of menu cost. The reader familiar with the argument can go directly to the next subsection. Markets M_1 and M_2 are identical in all respect but the level of willingness to pay. To simplify, we assume without loss of generality that the marginal cost of production is zero and we consider unit market sizes. Using our notations, $M_1 = (v_1, 0, m, 1)$ and $M_2 = (v_2, 0, m, 1)$. To solve the game we first solve for the equilibrium price in the second stage under both uniform pricing and price discrimination. Under price discrimination, firms A and B set a price p_i in market i equal to

$$p_i^D = \frac{2v_i}{m+4} \tag{149}$$

Firms charge more in the markets with higher willingness to pay: If $v_1 > v_2$ then $p_1^D > p_2^D$. Firms' profits in market i are

$$\pi_i^D = v_i^2 \frac{2+m}{(m+4)^2} \tag{150}$$

Under uniform pricing, firm A's demand in market $M_{1,2}$ is

$$q_A = \frac{v_1+v_2}{2} - p_A\left(1+\frac{m}{2}\right) + \frac{m}{2}p_B \tag{151}$$

Therefore market $M_{1,2} = (\frac{v_1+v_2}{2}, 0, m, 2)$. Firms treat the overall market as a single market of twice the size and with average willingness to pay $\frac{v_1+v_2}{2}$. Firms set a price p in markets M_1 and M_2 equal to

$$p^U = \frac{v_1+v_2}{m+4} \tag{152}$$

Firms' profits in market i are

$$\pi_i^U = \left(\frac{v_1+v_2}{2}\right)^2 \frac{2+m}{(m+4)^2} \tag{153}$$

Note that the uniform price is the average of the discriminatory prices

$$p^U = \frac{p_1^D + p_2^D}{2} \tag{154}$$

There is no effect of the first stage decision on the level of prices. Prices, however, vary under discriminatory pricing in a way that exploits the differences in willingness to pay. As

a consequences the sum of the profits under discriminatory pricing is greater than the profits under uniform pricing.

$$\pi^D > \pi^U \tag{155}$$

This result follows directly from the observation that the function $\pi(v) = v^2 \frac{2+m}{(m+4)^2}$is convex in v. Firms prefer to set two market specific prices than a single price for both markets because in equilibrium their pricing strategies exploit the market difference in a way that is consistent with the decisions they would make if they could collude on price in the second stage.

The result that firms never want to use uniform pricing when markets differ only by their level of willingness to pay applies to any number of markets. In fact, consider M_i markets for $i = 1..I$. The proof goes by contradiction. Assuming that price discrimination is not optimal implies that firms prefer to pool some markets (possibly all) together. Assume that markets $(i_1, .., i_J)$ are pooled together. This implies that $M_{i_1,..,i_J} \succ (M_{i_1,..,i_{J-1}}, M_{i_J})$. But this contradicts the above result. (This is not exactly correct since the derivation above assumed identical market sizes. The result, however, generalizes to any market size.)

Market Competition Consider the situation where markets M_1 and M_2 are identical in all respect but the level of market competition. As before, we assume without loss of generality that the marginal cost of production is zero and we consider unit market sizes. Using our notations, $M_1 = (v, 0, m_1, 1)$ and $M_2 = (v, 0, m_2, 1)$. As in the previous subsection, we first solve for the equilibrium price in the second stage under both uniform pricing and price discrimination. Under price discrimination, firms set a price p_i in market i equal to

$$p_i^D = \frac{2v}{m_i + 4} \tag{156}$$

Firms are more aggressive in the more competitive markets. If $m_1 > m_2$ then $p_1^D < p_2^D$. Firms profits in market i are

$$\pi_i^D = v^2 \frac{2 + m_i}{(m_i + 4)^2} \tag{157}$$

Under uniform pricing, firm A's demand in market $M_{1,2}$ is

$$q_A = v - p_A \left(1 + \frac{\frac{m_1+m_2}{2}}{2} \right) + \frac{\frac{m_1+m_2}{2}}{2} p_B \tag{158}$$

Therefore market $M_{1,2} = (v, 0, \frac{m_1+m_2}{2}, 2)$. Firms treat the overall market as a single market of twice the size and with the average level of market competition $m = \frac{m_1+m_2}{2}$. Firms set the same price p in market M_1 and M_2

$$p^U = \frac{2v}{\frac{m_1+m_2}{2} + 4} \tag{159}$$

Firms profits in market i are

$$\pi^U = v^2 \frac{2 + \frac{m_1+m_2}{2}}{\left(\frac{m_1+m_2}{2} + 4\right)^2} \tag{160}$$

The effect on profits of switching from uniform pricing to price discrimination depends on the values of m_1 and m_2. We can characterize the choice of pricing strategies in the first stage for some parameter values:

$$\pi^D > \pi^U \text{ if } m_1, m_2 > 2 \tag{161}$$

$$\pi^D < \pi^U \text{ if } m_1, m_2 < 2 \tag{162}$$

This result follows directly from the observation that the function $\pi(m) = v^2 \frac{2+m}{(m+4)^2}$ is convex in m for $m_2 > 2$ and concave in m for $m < 2$. For levels of market competition $m_1 < 2 < m_2$ uniform pricing may dominate or be dominated by price discrimination.

The intuition for this result is as follows. Switching from price discrimination to uniform pricing has two effects. The first effect is that prices are lower under uniform pricing,

$$p^U < \frac{p_1^D + p_2^D}{2} \tag{163}$$

To see that note that the function $p(m) = \frac{2v}{m+4}$ is convex in m. The second effect is that prices vary under price discrimination. However, price variation has a negative effect on joint profits because total firms' demand is the same in both markets

$$q_{1,A} + q_{1,B} = v - p_A - p_B \tag{164}$$

Because joint profits are concave in price, price variation decreases joint profits. In fact, firms always prefer to commit to a constant price over a varying price whose average is equal to the constant price. But this is exactly what uniform pricing gets firms to do: prices do not vary across markets. The relative impact of these two effects determines whether uniform pricing dominates, or is dominated by, price discrimination.

At this point it is important to distinguish the impact of varying prices under price discrimination in the case where markets differ in their level of willingness to pay and in the case where the differ in their level of market competition. When willingness to pay varies across markets, equilibrium prices vary under discriminatory pricing in a way that exploit the differences in willingness to pay. As a consequences the sum of the profits under discriminatory pricing is greater than the profits under uniform pricing. When the level of market competition varies across markets, prices vary under discriminatory pricing in a way that does not exploit willingness to pay across markets. Varying prices has a negative impact on profits.

We illustrate that uniform pricing and price discrimination may occur with two examples. To illustrate the use of uniform pricing, assume $m_1 = 0$ and $m_2 = 2$. The profits corresponding to these parameter values are $\pi^D = \frac{17}{72} < \frac{6}{25} = \pi^U$. To understand why it is not always optimal to use uniform pricing, consider the extreme two-market case where firm A and B are monopolist in market M_1 ($m_1 = 0$), and both firms compete fiercely in market M_2 (m_2 large). In that case, the pure strategy equilibrium under uniform pricing would be to set $p_1 = p_2$ close to zero in both markets. Firms are better off under price discrimination.

More generally, one could extend the analysis of market competition to multiple markets that differ only by their level of market competition. Firms would tend to pool together markets where competition is not too strong.

Some of the result are specific to the demand specification used. More specifically, the possibility of existence of uniform pricing is more general than what is suggested by the demand specification used in this analysis. To illustrate this point consider the finding that firms do not want to use uniform pricing when they sell in a monopolized market and in a fiercely competitive market. (See example above.) Consider the following alternative demand specification. As before, each firm is a monopolist in the first market. In addition to their monopolized market, firms compete a la Bertrand in the second market. Under the demand specification used before firms would not want to use uniform pricing. Assume now that firms' demand in their monopolized market is very price sensitive. If firms lower prices below the monopoly price, they loose a substantial share of demand. The equilibrium under uniform pricing is to set the same price in the competitive market as in the monopolized market. A commitment to use uniform pricing increases profits, at least as long as there is a positive demand in the competitive market at that the monopoly price. To summarize, this example shows that the relative advantage of uniform pricing over price discrimination depends on the level of the uniform price and this depends on how the price elasticity changes when firms

pool two markets together.

Marginal Cost Consider the situation where markets M_1 and M_2 are identical in all respect but the level of marginal cost. As before, we consider without loss of generality unit market sizes. Using our notations, $M_1 = (v, c_1, m, 1)$ and $M_2 = (v, c_2, m, 1)$. As before, we first solve for the equilibrium price in the second stage under both uniform pricing and price discrimination. Under price discrimination, firms set a price p_i in market i equal to

$$p_i^D = \frac{2\left(v + c_i\left(1 + \frac{m}{2}\right)\right)}{m+4} \tag{165}$$

Firms charge more in the markets with higher marginal cost. Firms' profits in market i are

$$\pi_i^D = (v - c_i)^2 \frac{2+m}{(m+4)^2} \tag{166}$$

Under uniform pricing, firm A's demand in market $M_{1,2}$ is

$$q_A = v - p_A\left(1 + \frac{m}{2}\right) + \frac{m}{2}p_B \tag{167}$$

Firms treat the overall market as a single market of twice the size. The profits of firm A in market $M_{1,2}$ are

$$\pi_{1,2,A} = \left(p_A - \frac{c_1 + c_2}{2}\right)\left(v - p_A\left(1 + \frac{m}{2}\right) + \frac{m}{2}p_B\right) \tag{168}$$

These profits correspond to the profits firms would earn in market $\hat{M}_{1,2} = (v, \frac{c_1+c_2}{2}, m, 2)$. The equilibrium price and profits in $M_{1,2}$ are the same as the price and profits that would result if firms would compete in a market where the marginal cost would be equal to the average of the two markets' marginal costs $\hat{M}_{1,2}$.Using the same results as before, firms set a price p in market 1 and 2 equal to

$$p^U = \frac{2\left(v + \frac{c_1+c_2}{2}\left(1 + \frac{m}{2}\right)\right)}{m+4} \tag{169}$$

Firms' profits in market i are

$$\pi_i^U = \left(v - \frac{c_1 + c_2}{2}\right)^2 \frac{2+m}{(m+4)^2} \tag{170}$$

The uniform price is the average of the discriminatory prices

$$p^U = \frac{p_1^D + p_2^D}{2} \tag{171}$$

Prices vary under discriminatory pricing in a way that exploit differences in cost across markets. As a consequences the sum of the profits under discriminatory pricing is greater than the profits under uniform pricing.

$$\pi^D > \pi^U \tag{172}$$

To prove this result, note that the function $\pi(c) = (v-c)^2 \frac{2+m}{(m+4)^2}$ is convex in c.

Market Size Consider the situation where markets M_1 and M_2 are identical in all respect but their sizes. Using our notations, $M_1 = (v, c, m, s_1)$ and $M_2 = (v, c, m, s_2)$. Under price discrimination, firms set a price p_i in market i equal to

$$p_i^D = \frac{2v}{m+4} \tag{173}$$

Firms' prices are independent of market size. Firms do not discriminate under price discrimination. This result is fairly intuitive. There are no intrinsic differences between markets. The finding that firms use uniform pricing is not related to collusive behavior. A monopolist ($m = 0$) would also choose not to discriminate. For this reason, we treat this case of uniform pricing as trivial.

Summary To summarize, we identify three effects of switching from price discrimination to uniform pricing:

1. Price effect: The level of price may differ under price discrimination and uniform pricing. In fact, we have shown that firms set lower prices under uniform pricing when markets differ only by their level of market competition.

2. Market Customizing Effect: Firms may be able to adjust prices to the characteristics of market under price discrimination in a way that is consistent with the decision they would make if they could collude in the second stage of the game.

3. Price Noise Effect: Firm may vary prices across markets in a way that is not consistent with the decision they would make if they could collude in the second stage of the game. In that case, firms would prefer to commit not to vary prices if they could.

In the case of differences in willingness to pay or differences in cost, only the market customizing effect plays a role with the consequence that price discrimination is optimal. In the case of differences in the level of market competition, both the price effect and the price noise effect play a role and the relative strength of these two effects determines whether price discrimination or uniform pricing is preferred. To summarize, the analysis reveals that price categories may emerge only if markets differ in terms of their level of competitiveness.

6.3.3 Price categories

We find that uniform pricing is optimal only when markets differ in their level of market competition. An implication is that price categories will emerge only if some markets differ in their level of market competition. We provide three examples where price categories will emerge. In the first example, there are 3 markets with three different levels of market competition. Firms choose to pool the two markets with low levels of market competition. In the second example, markets differ both in terms of market competition and willingness to pay. Firms pool markets with different levels of market competition but not those with different levels of willingness to pay. The last example has four markets and firms offer two price categories in equilibrium.

Price categories with only Three Levels of Market Competition Consider the following 3 markets: $M_1 = (v, 0, m_1, \frac{1}{2})$, $M_2 = (v, 0, m_2, \frac{1}{2})$ and $M_3 = (v, 0, m_3, 1)$ and assume that $1 < m_1 < m_2 < 2 < m_3$. The analysis of market competition implies that $M_{1,2} \succ (M_1, M_2)$ and it follows that price discrimination is dominated by price categories $(M_{1,2}, M_3) \succ (M_1, M_2, M_3)$. In addition since $M_{1,2} = (v, 0, \frac{m_1+m_2}{2}, 1)$ with $\frac{m_1+m_2}{2} > 2$, the same analysis of market competition implies that uniform pricing is dominated by price categories $(M_{1,2}, M_3) \succ M_{1,2,3}$. It follows that price category outcome $(M_{1,2}, M_3)$ is the unique equilibrium in the first stage.

Price categories with only Two Levels of Market Competition Consider the following 3 markets: $M_1 = (v_1, 0, m_1, \frac{1}{2})$, $M_2 = (v_1, 0, m_2, \frac{1}{2})$ and $M_3 = (v_2, 0, \frac{m_1+m_2}{2}, 1)$ and assume that $m_1, m_2 < 1$. The analysis of market competition implies that $M_{1,2} \succ (M_1, M_2)$ and it follows that price discrimination is dominated by price categories $(M_{1,2}, M_3) \succ (M_1, M_2, M_3)$. In addition since $M_{1,2} = (v_1, 0, \frac{m_1+m_2}{2}, 1)$, the analysis of willingness to pay implies that uniform pricing is dominated by price categories $(M_{1,2}, M_3) \succ M_{1,2,3}$. It follows that price category outcome $(M_{1,2}, M_3)$ is the unique equilibrium in the first stage. Although $M_{1,2}$ and

M_3 have low levels of market competition, it is not optimal to pull them together because these two markets vary in terms of willingness to pay.

Multiple price categories Consider the following 4 markets: $M_1 = (v_1, 0, m_1, \frac{1}{2})$, $M_2 = (v_1, 0, m_2, \frac{1}{2})$, $M_3 = (v_2, 0, m_1, \frac{1}{2})$, $M_4 = (v_2, 0, m_2, \frac{1}{2})$, and assume that $m_1, m_2 < 2$. Again, $M_{1,2} \succ (M_1, M_2)$ and similarly $M_{3,4} \succ (M_3, M_4)$. The analysis of willingness to pay implies that $(M_{1,2}, M_{3,4}) \succ M_{1,2,3,4}$. Considering all possible market combinations reveal that price category outcome $(M_{1,2}, M_{3,4})$ is the unique equilibrium in the first stage.

6.4 Search Cost and Price Categories

In this Section, we present a simple model with search costs showing that competing firms may offer price categories. The model explains why firms may sell a set of products at the same price, although these products have different production costs and consumers are willing to pay different amounts for the products. We recognize that the analysis is sensitive to the assumptions of the model. Therefore, the model should be interpreted with care. The point is to illustrate the possibility that price categories may endogenously emerge in an equilibrium model with search costs.

Price categories arise when some consumers are imperfectly informed about their preferences and others are perfectly informed. Those who are imperfectly informed may benefit from search because doing so allows them to find a better match. We show that firms perfectly price discriminate when there are no imperfectly informed consumers. When there are some imperfectly informed consumers, however, price categories are offered in equilibrium to eliminate consumer search. The intuition is that consumers have an incentive to shop around when firms price discriminate. The presence of uninformed consumers forces firms to offer all products at the same price.

The analysis proceeds in two steps. Proposition 1 identifies a set of characteristics on consumer preferences that implies that the only equilibrium can be either uniform pricing or price discrimination. Under uniform pricing, firms sell all products at the same price although these products have different production cost and demands. Under price discrimination, firms sell all products at different prices. Proposition 2 identifies a set of characteristics on consumers preferences that implies that price categories emerge as the only equilibrium where a price category equilibrium is defined as an outcome where some products are sold at the same price while other products are sold at a different price.

6.4.1 The Model

We expose the model within a shopping context. A store offers I different products. Product i costs c_i to manufacture. There is a mass one of consumers. Each consumer values only one of the I products. There are two types of consumers. Informed consumers know which product they like. Fraction f_i of consumers are informed and are willing to pay v_1 for product i. We assume without loss of generality that $v_1 > ... > v_I$. There are also $f_v = 1 - \sum_i f_i$ uninformed consumers. Uninformed consumers also like only one product. They are willing to pay v_H with probability $f_{v,H}$, and v_L with probability $f_{v,L}$ with $f_{v,H} + f_{v,L} = f_v$ and $v_H > v_1 > v_I > v_L > c_i$, for $i = 1 \ldots I$. Uninformed consumers know their willingness to

pay but they do not know which product they like until they go to the store. After a 'touch and feel' experience at the store, uninformed consumers find out which product they like. An uninformed consumers likes product i with probability g_i with $\sum_i g_i = 1$. We assume for now that $g_i > 0$.

When we consider the model with two stores we will assume that the products offered by both stores are not exactly identical in the eyes of uninformed consumers in the following sense. We assume that the product that an uninformed consumer likes at one store may not be the same as the product s/he likes at the other store. This is reasonable if two products of type i may look almost identical to some consumers (the informed ones) but quite different to other consumers (the uninformed). To simplify, we assume that the events that an uninformed consumer likes product i from one firm and product j from another are independent. Finally, we make two assumptions:

$$(A1) \quad f_{v,H} g_i (v_H - v_i) < f_i (v_i - c)$$
$$(A2) \quad (f_{v,H} g_i - f_i)(v_i - v_L) < f_{v,L} g_i (v_L - c)$$

These assumptions hold as long as there are not too many uninformed consumers or if the v_H-uninformed consumers are willing to pay only a small premium above informed consumers and the v_L-uninformed consumers are willing to pay large discount below informed consumers. Without these assumptions, it would be optimal for a monopolist to offer price categories rather than price discriminate informed consumers. Imperfect information alone would explain the existence of price category. These assumptions allow us to focus on strategic reasons for price categories.

6.4.2 Analysis

We study whether price categories are offered under different market structures. We consider perfect competition, monopoly, and imperfect competition.

Perfect Competition Assume free entry and perfect competition. In equilibrium, firms sell product i at $p_i = c_i$. There are no price categories as long as costs differ across products.

Monopoly Pricing The monopolist sets the price of good i at $p_i = v_i$. This pricing strategy maximize profits. To see that, consider the alternative strategy where the monopolist sell only to v_H-uninformed consumers. The price of product i is $p_i = v_H$. Assumption (A1) says that it is optimal to deviate and to sell product $i$to informed consumers at $p_i = v_i$.

Consider the case where the monopolist sell to all consumers. The price of product i is at $p_i = v_L$. Assumption (A2) says that it is optimal to deviate and to increase the price of product i to $p_i = v_i$. Under monopoly pricing, there are no price categories as long as informed consumers are willing to pay different amounts for different products.

Imperfect Competition We borrow Diamond (1971) model of imperfect competition. See also Lal and Matutes (1994), Anderson and Renault (1999) and Verboven (1999). In this class of models, imperfect competition is caused by (arbitrarily small) search costs. To simplify, we assume that there are two stores, A and B. In addition, we assume that each store offers all the products.

Stores compete as follows. They first set a price for each product, $p_{s,i}$, for $s = A, B$ and $i = 1 \dots I$. They cannot change this price afterwards. Consumers choose which store to join. They do not know a store's set of prices before joining the store. Once at the store, say store A, a consumer observes $p_{A,i}$ for $i = 1 \dots I$ and can buy a product from that store or join the other store. If the consumer goes to the other store, s/he has to pay an arbitrarily small search cost ε. After visiting the other store, store B, the consumer can buy there, return to A (at no additional cost) and buy from A, or not buy at all. Assuming that consumers pay the search cost only if they visit a second store simplifies the presentation. This assumption is reasonable, for example, if consumers have to go to one store anyway, because they have to shop for other needs that can be fulfilled in a single visit. The search cost captures the cost of visiting an additional store and searching again for products $i = 1 \dots I$.

We define a shopping strategy as the decision of which store to join initially, whether to buy or to visit the other store conditional on the set of prices posted at the first store visited, and conditional on visiting the second store, the final decision to buy conditional on all prices. A consumer who is indifferent between joining either store will randomly choose a store (with equal probability), and similarly if the consumer is indifferent between several products after visiting both stores.

We focus on pure strategy subgame perfect Nash equilibrium. In addition, we impose that all players must have rational expectations in equilibrium. Consumers form a belief on firms' prices that is rational in equilibrium in the sense that it is consistent with the firms' actual prices. Consumers select their shopping strategies according to their beliefs. In addition, firms cannot benefit by changing their prices given consumer shopping strategies.

Proposition 5 *There is a unique equilibrium. If $f_v = 0$, both stores charge $p_i = v_i$. If $f_v > 0$, both firms charge $p_i = v_L$.*

Proof. Consider the case $f_v = 0$ and suppose firms charge $p_i = v_i$. Informed consumers can visit one store and buy the product they like at that store. Alternatively, they can visit both stores. They get the same consumption utility if they visit both store but they have to pay the additional search cost ε. Therefore, it is optimal to visit a single store. Consider firms' pricing decisions. If a firm sets the price of good i at $p > v_i$, informed consumers who like good i do not buy. The store's profits decreases. What about decreasing the price of a product? This would not change consumers' expectations. Quantities sold would remain constant and profits would decrease again.

Next, we show that $p_i = v_i$ is the only equilibrium. The prices of good i have to be equal in any equilibrium, $p_{i,A} = p_{i,B}$, for $i = 1 \ldots I$. Otherwise, all informed consumers of type i would visit only the store that offers the lowest price. Assume $p_{i,A} = p_{i,B} > v_i$. Consumers who like good i visit a single store and do not buy. Firms are better off reducing the price to v_i since this triggers consumers of type i to buy. Assume $p_{i,A} = p_{i,B} < v_i$. Each firm has an incentive to increase the price by at most ε. Consumers still buy and profits increase.

Consider the case $f_v > 0$ and suppose firms charge $p_i = v_L$. As before, visiting an additional store does not increase consumption utility and reduces overall utility by ε. Consumers visit a single store and buy the product they like at that store. Next, we show that firms have no incentive to change prices. If a firm increases the price of good i by less than ε, that firm looses the uninformed consumers who like that product and who are willing to pay only v_L. For ε small enough, $\varepsilon(f_{v,H} g_i + f_i) < f_{v,L} g_i (v_L - c)$ and profits decrease. Firms do not gain by lowering prices either since this does not change consumers' shopping strategy.

Finally, we show that $p_i = v_1$ is the only equilibrium. A1 implies that $p_i \leq v_i$ in any equilibrium. For the same reasons as before, $p_{i,A} = p_{i,B}$ in any equilibrium. Any price $v_L < p_i < v_i$ cannot be an equilibrium because firms have an incentive to increase p_i by ε. Consider pricing strategy $p_{i,A} = p_{i,B} = v_i$. For ε small enough $\varepsilon < g_1(p_{2,A} - p_{1,B}) = g_1(v_2 - v_1)$. Any uninformed consumer who likes product 2 in store A will shop to store B. Assume that consumer likes again product 2 in store B. Then, he will choose randomly between store A and B. But either store could attract that consumer for sure by slightly lowering the price of product 2. Therefore, $p_{i,A} = p_{i,B} = v_i$ cannot be an equilibrium. ■

The first result is not new. Diamond (1971) showed that when consumers incur even a small search cost to learn firms' prices, the equilibrium prices of otherwise homogeneous goods are the monopoly prices. The second result is new. It says that the presence of an arbitrarily small fraction of uninformed consumers destroys the fully collusive equilibrium where firms perfectly price discriminate. With the presence of uninformed consumers, all firms charges a

uniform price. Under uniform prices, uninformed consumers have no incentive to search and an equilibrium is reached again.

The role of uninformed consumers is now clear. The presence of uninformed consumers with high valuations implies that full price discrimination cannot be an equilibrium. The presence of uninformed consumers with low valuations implies that firms do not want to deviate from the low price equilibrium.

The important point is that price categories do not emerge under perfect competition or monopoly pricing. When there are no uninformed consumers, price categories do not emerge under imperfect competition either.

The existence of uniform pricing rests on two essential conditions: $f_v > 0$ and $\varepsilon > 0$. Uniform pricing is the only equilibrium for any arbitrarily small fraction of uninformed consumers $f_v > 0$. The problem with uninformed consumers is that they search if there are price differences. This implies that firms cannot price discriminate when $f_v > 0$. The second condition is $\varepsilon > 0$. Positive search cost is the source of market power. Interestingly, uniform pricing is not an equilibrium if some uninformed consumers have zero search cost ($\varepsilon = 0$). In that case, the only equilibrium is marginal cost pricing.

6.4.3 Price categories

So far, we have shown that products were either sold at different prices (price discrimination) or at the same price (uniform pricing). In this section, we show that price categories can emerge in equilibrium. We define a price category equilibrium as an outcome where some products are sold at the same price while other products are sold at a different price. Define J as a subset of products and J^c its complement ($J \cup J^c = I$) such that both J and J^c are non-empty.

Proposition 6 *If $f_v > 0$ and $g_i = 0$ for $i \in J$, then the only equilibrium is a price category equilibrium with $p_i = v_L$ for $i \in J$ and $p_i = v_i$ for $i \in J^c$.*

Proof. Since no uninformed consumer like product $i \in J^c$, charging $p_i = v_i$ is an equilibrium. A similar argument as the one presented in the proof of proposition 1 shows that this is the only equilibrium. ■

We conclude with a two final comments:

(1) One can easily change consumer preferences to develop a more general model where multiple categories are offered in equilibrium.

(2) As the previous literature with search cost, the results presented here clearly generalize when there are more than 2 stores.

7 References

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